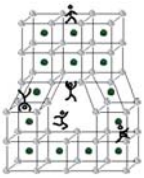
A Step Toward Devices



Monday,
Nov. 10 2003

MATS275: INTRODUCTION TO MATERIALS SCIENCE

- Semiconductor Statistics
  - The Fermi-Dirac Distribution
  - Density of States
  - Number of Conduction Electrons
- Intrinsic vs. Extrinsic
- Dependence of E_F on Doping
- pn Junctions

Filling Levels

- Recall,number of electrons between E and E+dE:

$$dN^* = N(E)dE$$

- Where N(E) is the population density:

$$N(E) = 2Z(E)F(E)$$

- And Z(E) is the density of states:

$$Z(E) = \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}$$

- And F(E) is the Fermi distribution function:

$$F(E) = \frac{1}{e^{(E-E_F)/kT}+1} \approx e^{\frac{-(E-E_F)}{kT}}$$

Population Density

- The density of electrons is:

$$N_e = \frac{1}{V}\int_0^\infty 2Z(E)F(E)dE$$

- So the number in the conduction band is...

$$N_c = \frac{1}{V}\int_0^\infty N(E)dE = \frac{1}{V}\int_0^\infty 2Z(E)F(E)dE$$

$$= \int_0^\infty 2\left[\frac{1}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E-E_c}\right]\left[\frac{1}{e^{\frac{(E-E_F)}{kT}}+1}\right]dE$$

Population Density

- Make the exponents dimensionless and use E_c as the origin…

$$\varepsilon = \frac{(E - E_c)}{kT} \qquad \eta = \frac{(E_F - E_c)}{kT}$$

$$N_e = \frac{1}{2\pi^2}\left(\frac{2mkT}{\hbar^2}\right)^{3/2} \int_0^\infty \frac{\varepsilon^{1/2}}{e^{(\varepsilon-\eta)}+1} d\varepsilon \equiv N_c \mathcal{F}_{1/2}(\eta)$$

$$N_c \equiv \frac{1}{4}\left(\frac{2mkT}{\pi\hbar^2}\right)^{3/2} \qquad \mathcal{F}_{1/2}(\eta) \equiv \frac{2}{\pi^{1/2}} \int_0^\infty \frac{\varepsilon^{1/2}}{e^{(\varepsilon-\eta)}+1} d\varepsilon$$

Effective Density of States

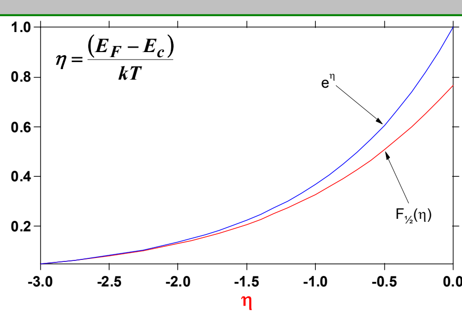
- The conduction band "effective density of states" is

$$N_c \equiv \frac{1}{4}\left(\frac{2m_e^* kT}{\pi\hbar^2}\right)^{3/2} = \left(4.84 \times 10^{15} \frac{1}{cm^3 K^{3/2}}\right)\left(\frac{m_e^*}{m_0}\right)^{3/2} T^{3/2}$$

- Similarly, for the valence band…

$$N_v \equiv \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$$

The Fermi Integral



The Carrier Density

- What if η<<0 (i.e., if the conduction band edge is well above the Fermi level)?

$$\mathcal{F}_{1/2}(\eta) \equiv \frac{2}{\pi^{1/2}} \int_0^\infty \frac{\varepsilon^{1/2}}{e^{(\varepsilon-\eta)}+1} d\varepsilon \cong e^{\eta}$$

$$N_e = N_c \mathcal{F}_{1/2}(\eta) \cong N_c e^{\eta} = \frac{1}{4}\left(\frac{2m_e^* kT}{\pi\hbar^2}\right)^{3/2} e^{-\frac{E_c - E_F}{kT}}$$

$$= \left(4.84\times10^{15}\frac{1}{cm^3 K^{3/2}}\right)\left(\frac{m_e^*}{m_0}\right)^{3/2} T^{3/2} e^{-\frac{E_g}{2kT}}$$

The Intrinsic Concentration

- Similarly…

$$N_e = N_c e^{-\frac{E_c - E_F}{kT}} \qquad N_h = N_v e^{-\frac{E_F - E_v}{kT}}$$

$$N_c \equiv \frac{1}{4}\left(\frac{2m_e^* kT}{\pi\hbar^2}\right)^{3/2} \qquad N_v \equiv \frac{1}{4}\left(\frac{2m_h^* kT}{\pi\hbar^2}\right)^{3/2}$$

$$n_i^2 = N_e N_h = N_c N_v e^{-\frac{E_c - E_F}{kT}} e^{-\frac{E_F - E_v}{kT}} = N_c N_v e^{-\frac{E_g}{kT}}$$

$$n_i = \sqrt{N_c N_v} e^{-\frac{\left(E_g/2\right)}{kT}}$$

The Intrinsic Concentration

- Since every electron leaves a hole…

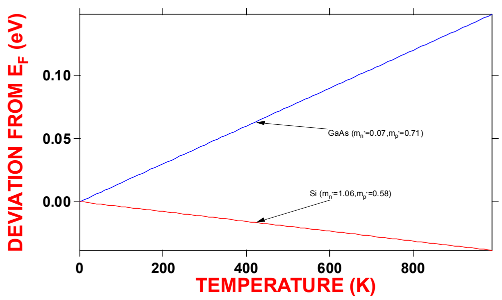
$$\sqrt{N_c N_v} e^{-\frac{\left(E_g/2\right)}{kT}} = N_c e^{-\frac{E_c - E_F}{kT}}$$

$$\sqrt{\frac{N_v}{N_c}} = exp\left(-\frac{E_c - E_F - \frac{1}{2}E_g}{kT}\right)$$

$$E_c - E_F + \frac{1}{2}E_g = \frac{1}{2}kT\,ln\left(\frac{N_c}{N_v}\right)$$

$$E_F = E_c - \frac{1}{2}E_g - \frac{1}{2}kT\,ln\left(\frac{N_c}{N_v}\right)$$

Deviation of E_F with T



Donor Doping

- If an atom with a valence of +1 compared to the host is added to a substitutional site, the extra electron is weakly bound to the atom:

$$E_D = -\frac{1}{2}\frac{e^4 m_e^*}{(4\pi\kappa\varepsilon_0\hbar)^2} = -\frac{1}{2}\frac{e^4 m_0}{(4\pi\varepsilon_0\hbar)^2}\frac{1}{\kappa^2}\frac{m_e^*}{m_0} = (-13.6\,\mathrm{eV})\frac{1}{\kappa^2}\frac{m_e^*}{m_0}$$

- Ex., P in Si

$$E_D = (-13.6\,\mathrm{eV})\frac{1}{\kappa^2}\frac{m_e^*}{m_0}$$

$$= (-13.6\,\mathrm{eV})\frac{1}{(11.7)^2}(1.06) = -0.105\,\mathrm{eV}$$

Typical Donors & Acceptors

- Si (1.12 eV)

(p) B (45 meV), Al (67 meV), Ga (72 meV), In (160 meV)

(n) P (45 meV), As (54 meV), Sb (39 meV)

- Ge (0.66 eV)

(p) B (10 meV), Al (10 meV), Ga (11 meV), In (11 meV)

(n) P (12 meV), As (13 meV), Sb (9.6 meV)

- GaAs (1.42 eV)

(p) C (26), Be (28), Mg (28), Zn (31), Si (35)

(n) Si (5.8 meV), Ge (6 meV), S (6 meV)

Extrinsic Semiconductors

- Recall that n_i was a constant:

$$n_i^2 = N_e N_h = N_c N_v e^{-\frac{E_g}{kT}}$$

- If we increase N_e, N_h must go down.
- Also must have charge neutrality:

$$N_e^- - N_d^+ = N_h^+ - N_a^-$$
$$N_e^- + N_a^- = N_d^+ + N_h^+$$

Changing the Fermi Level

- The Fermi level must adjust to preserve charge neutrality: $N_e^- - N_h^+ = N_d^+$

$$N_e = N_c e^{-\frac{E_c - E_F}{kT}}$$

$$N_h = N_v e^{-\frac{E_F - E_v}{kT}}$$

$$N_d^+ = N_d \left[1 - \underbrace{\frac{1}{e^{\frac{E_d - E_F}{kT}} + 1}}_{\text{f(E) for an electron in donor level}} \right]$$

Finding the Fermi Level

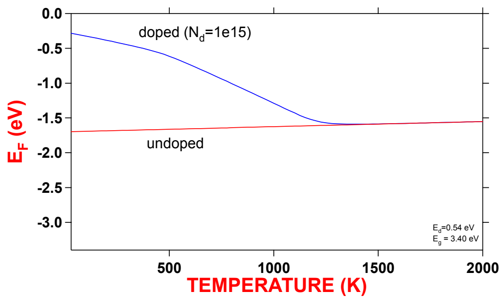
$$N_d^+ = N_d \left[1 - \frac{1}{e^{\frac{E_d - E_F}{kT}} + 1} \right] = N_d \frac{1}{1 + e^{\frac{E_F - E_d}{kT}}}$$

$$N_e = N_d^+ + N_h$$

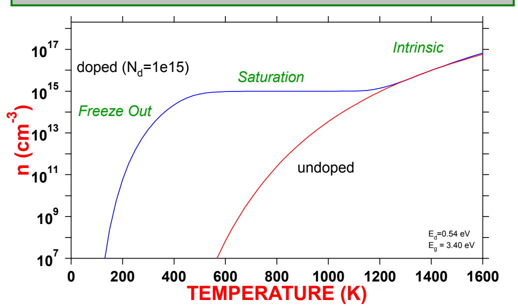
$$N_c e^{-\frac{E_c - E_F}{kT}} = N_d \frac{1}{1 + e^{\frac{E_F - E_d}{kT}}} + N_v e^{-\frac{E_F - E_v}{kT}}$$

Solve this for E_F and plug back in to get n...

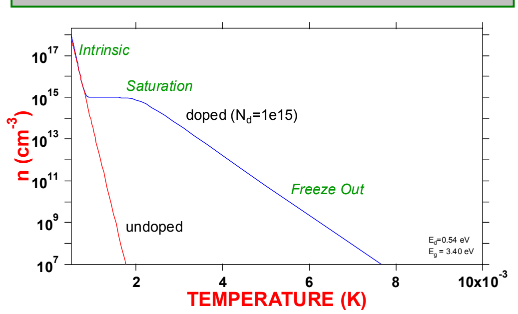
A Wide-Gap Semiconductor



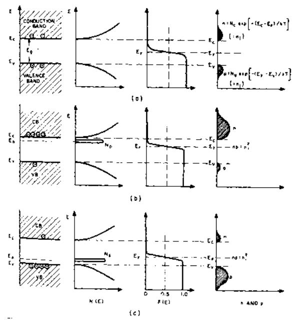
A Wide-Gap Semiconductor



A Wide-Gap Semiconductor

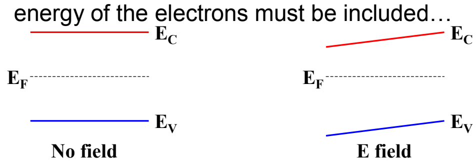


Changing the Fermi Level

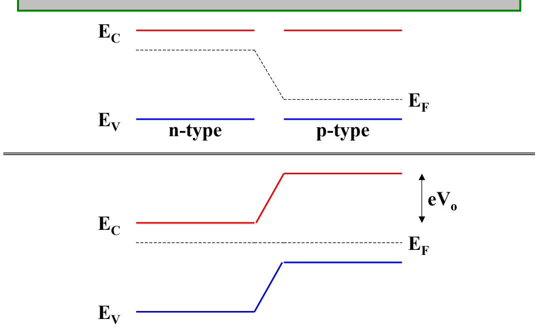


The Effect of an Applied Field

- If an electric field is applied, the potential energy of the electrons must be included…



The pn Junction



The pn Junction

- If the hole concentration on the p side is…

$$N_h = N_v e^{-\frac{E_F - E_v}{kT}} \Rightarrow E_F - E_v = kT\, ln\left(\frac{N_v}{N_h}\right)$$

- And on the n side is…

$$N_e = N_c e^{-\frac{E_c - E_F}{kT}} \Rightarrow E_c - E_F = kT\, ln\left(\frac{N_c}{N_e}\right)$$

- Put these together

$$\left(E_{F_p} - E_v\right) + \left(E_c - E_{F_n}\right) = kT\, ln\left(\frac{N_v}{N_h}\right) + kT\, ln\left(\frac{N_c}{N_e}\right)$$

$$E_g - \left(E_{F_n} - E_{F_p}\right) = kT\, ln\left(\frac{N_v N_c}{N_a N_d}\right)$$

The pn Junction

- So that the contact potential V_0 is…

$$E_g - (eV_0) = kT\, ln\left(\frac{N_v N_c}{N_a N_d}\right)$$

$$V_0 = \frac{E_g}{e} + \frac{kT}{e} ln\left(\frac{N_a N_d}{N_v N_c}\right)$$

$$n_i^2 = N_c N_v e^{-\frac{E_g}{kT}}$$

$$ln\left(n_i^2\right) = ln\left(N_c N_v\right) - \frac{E_g}{kT}$$

$$\frac{E_g}{e} = \frac{kT}{e} ln\left(\frac{N_v N_c}{n_i^2}\right)$$

$$V_0 = \frac{kT}{e} ln\left(\frac{N_a N_d}{n_i^2}\right)$$

A Si pn-Junction

- Consider a pn-junction with $N_a=10^{17}$ cm^{-3} and $N_d=10^{16}$cm^{-3}:

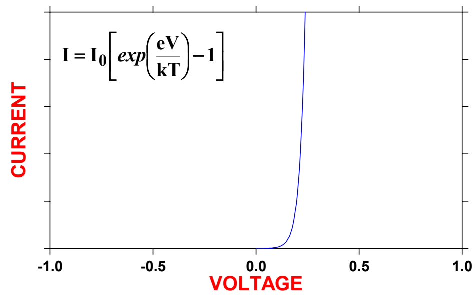
$$n_i = 1.5\times 10^{10}\,\text{cm}^{-3}$$

$$V_0 = \frac{kT}{e} ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$V_0 = 300\text{K}\,\frac{1.38\times 10^{-23}\,\frac{\text{J}}{\text{K}}}{1.602\times 10^{-19}\,\text{C}}\, ln\left[\frac{\left(10^{17}\,\text{cm}^{-3}\right)\left(10^{16}\,\text{cm}^{-3}\right)}{\left(1.5\times 10^{10}\,\text{cm}^{-3}\right)^2}\right]$$

$$V_0 = 300\text{K}\left(8.614\times 10^{-5}\,\text{eV/K}\right)(29.12) = 0.752\,\text{V}$$

The Diode Equation



The Zener Diode

- If we heavily reverse bias, we can get tunneling of the electrons…