Physics 260
Dr. Ingham

Name Key
November 22, 2002

HOUR TEST #3

HONOR PLEDGE:

In completing this test, I have neither given nor received unauthorized assistance.

(signature)

- This is a closed-book, closed-notes test. No written material is allowed.
- You are permitted to use an electronic calculator only to assist with numerical work.
- Assume that the given quantities are accurate enough to justify three (and only three) figures in your final answers.
- **Show your work, especially on the problems!** Credit, especially partial credit on problems where your final answer is incorrect, will depend on the work shown.

The test consists of 15 questions and 2 problems. Perfect score is 100.
Questions 1-15 count 4 points each. (60%)
Problems 1-2 count 20 points each. (40%)

Possibly useful information.

terrestrial acceleration due to gravity: $g = 9.80\ m/s^2$

$\pi = 3.14159265$ $e = 2.71828183$ $\log_{10}e = 0.43429448$

$1\ eV = 1.602 \times 10^{-19}\ J$ $1\ year = 3.156 \times 10^7\ sec$

astronomical unit (AU) = $1.496 \times 10^{11}\ m$

speed of light in vacuum:	$c = 2.998 \times 10^8\ m/s$
constant of gravitation:	$G = 6.673 \times 10^{-11}\ N\text{-}m^2/kg^2$
Planck's constant:	$h = 6.626 \times 10^{-34}\ J\text{-}s = 4.136 \times 10^{-15}\ eV\text{-}s$
"h-bar"	$\hbar \equiv h/2\pi = 1.055 \times 10^{-34}\ J\text{-}s = 6.583 \times 10^{-16}\ eV\text{-}s$
Boltzmann constant:	$k = 1.381 \times 10^{-23}\ J/K = 8.617 \times 10^{-5}\ eV/K$
Avogadro's number	$N_A = 6.022 \times 10^{23}$ particles/mole
Universal gas constant:	$R = N_A k = 8.315\ J/(mol\text{-}K)$
Stefan-Boltzmann constant:	$\sigma = 5.671 \times 10^{-8}\ W/(m^2K^4)$
unified mass unit (dalton):	$(1\ u) = 1.661 \times 10^{-27}\ kg$
energy equivalent of 1 dalton:	$(1u)c^2 = 1.493 \times 10^{-10}\ J = 931.9\ MeV$
proton (rest) mass:	$m_p = 1.673 \times 10^{-27}\ kg$
rest energy of proton:	$m_pc^2 = 1.504 \times 10^{-10}\ J = 938.6\ MeV$
electron (rest) mass:	$m_e = 9.109 \times 10^{-31}\ kg$
rest energy of electron:	$m_ec^2 = 9.187 \times 10^{-14}\ J = 0.5111\ MeV$
elementary unit of charge:	$e = 1.602 \times 10^{-19}\ C$
product of h and c:	$hc = 1.987 \times 10^{-25}\ J\text{-}m = 1.240 \times 10^{-6}\ eV\text{-}m = 1240\ eV\text{-}nm$
standard temperature:	$T = 0\ ^{\circ}C = 273.15\ K$
standard pressure:	$P = 1.000$ atmosphere $= 1.013 \times 10^5\ N/m^2$

Questions 1-15. (4 points each) On most of these questions, partial credit is not available. You are not required to show your work.

Q1. The electrostatic field just outside the surface of a conductor ________.

(A) is always zero
(B) is always parallel to the surface
(C) is always perpendicular to the surface
(D) cannot be directed outward if the overall charge on the conductor is zero

Q1. C

be written

Q2. The SI unit for electric field can ~~between~~ as 1 N/C (one newton per coulomb). This is equivalent to ____________.

(A) 1 V/m^2
(B) 1 V/m
(C) 1 V
(D) 1 V·m
(E) 1 $V·m^2$

Q2. B

Q3. Write a correct SI unit for electric dipole moment.

Q3. C·m or Coulomb-meters

Q4. How much electrostatic potential energy is stored when an initially "uncharged" 5.00-microfarad capacitor is "charged up" to a potential difference of 30.0 volts?

$P.E. = \frac{1}{2}CV^2 = \frac{1}{2}(5 \times 10^{-6})(300)^2 = 2.25 \times 10^{-3}$

Q4. 2.25×10^{-3} J

Q5. The magnitude of the electric field above a uniformly charged infinite horizontal sheet __________________ the distance above the sheet.

(A) varies in direct proportion to
(B) varies in inverse proportion to
(C) varies directly as the square of
(D) varies inversely as the square of
(E) is independent of
(F) varies directly as the logarithm of
(G) varies inversely as the logarithm of

Q5. E

Q6. Two capacitors (C_1 = 8.00 μF and C_2 = 6.00 μF) are connected in series in a circuit. What is the effective capacitance of this combination? Include correct units for your answer.

$C_{series} = \frac{C_1 C_2}{C_1 + C_2} = \frac{48}{14} \mu F = 3.43 \mu F$

Q6. 3.43 μF

Q7-8. A constant potential difference V_o is applied to a cylindrical resistor that has cross-sectional area A and total resistance R.

Q7. Write an expression for the power delivered to the resistor <u>in terms of given quantities</u>.

DC Power $= V_o I = V_o\left(\frac{V_o}{R}\right)$

Q7. V_o^2/R

Q8. Write an expression for the current density j in the resistor <u>in terms of given quantities</u>.

$j = \frac{I}{A} = \frac{V_o}{AR}$

Q8. V_o/AR

Q9. Carefully state Kirchoff's current law (KCL).

Q9. The sum of the currents leaving any node is zero.

Q10. A capacitor ($C = 25.0 \times 10^{-6}$ F) is initially "charged" to a potential difference of 50.0 volts. Then, beginning at time zero, the capacitor is "discharged" through a resistor of resistance R. The potential difference across the capacitor is found to vary with time according to $V_C(t) = (50.0 \text{ volts})e^{-\frac{t}{(0.200 \text{ sec})}}$. What is the value of the resistance? Include proper units.

$RC = 0.200 \text{ sec} \Rightarrow R = \frac{0.200 \text{ sec}}{25.0 \times 10^{-6} \text{ F}} = 8000\ \Omega$

Q10. $8000\ \Omega$

Q11. $\oiint_{\substack{closed \\ surface}} \vec{B} \cdot d\vec{S} =$

(A) 0
(B) Q_{net}/ε_o
(C) $\mu_o I_{conduction}$
(D) $\mu_o I_{displacment}$
(E) $\mu_o(I_{conduction}+I_{displacement})$

Q11. A

Q12. (True or False) In a propagating electromagnetic wave in vacuum, the electric field is always perpendicular to the magnetic field and one of the two fields is always parallel to the direction of propagation. X

Q12. False

Q13. Which of the following is **NOT** the correct form of one of Maxwell's equations in differential form?

(A) $\vec{\nabla} \cdot \vec{E} = \dfrac{\rho}{\varepsilon_o}$

(B) $\vec{\nabla} \cdot \vec{B} = 0$

(C) $\vec{\nabla} \times \vec{B} = \mu_o \vec{J} + \mu_o \varepsilon_o \dfrac{\partial \vec{E}}{\partial t}$

(D) $\vec{\nabla} \times \vec{E} = \mu_o \varepsilon_o \dfrac{\partial \vec{B}}{\partial t}$ $\ominus$ e

Q13. D

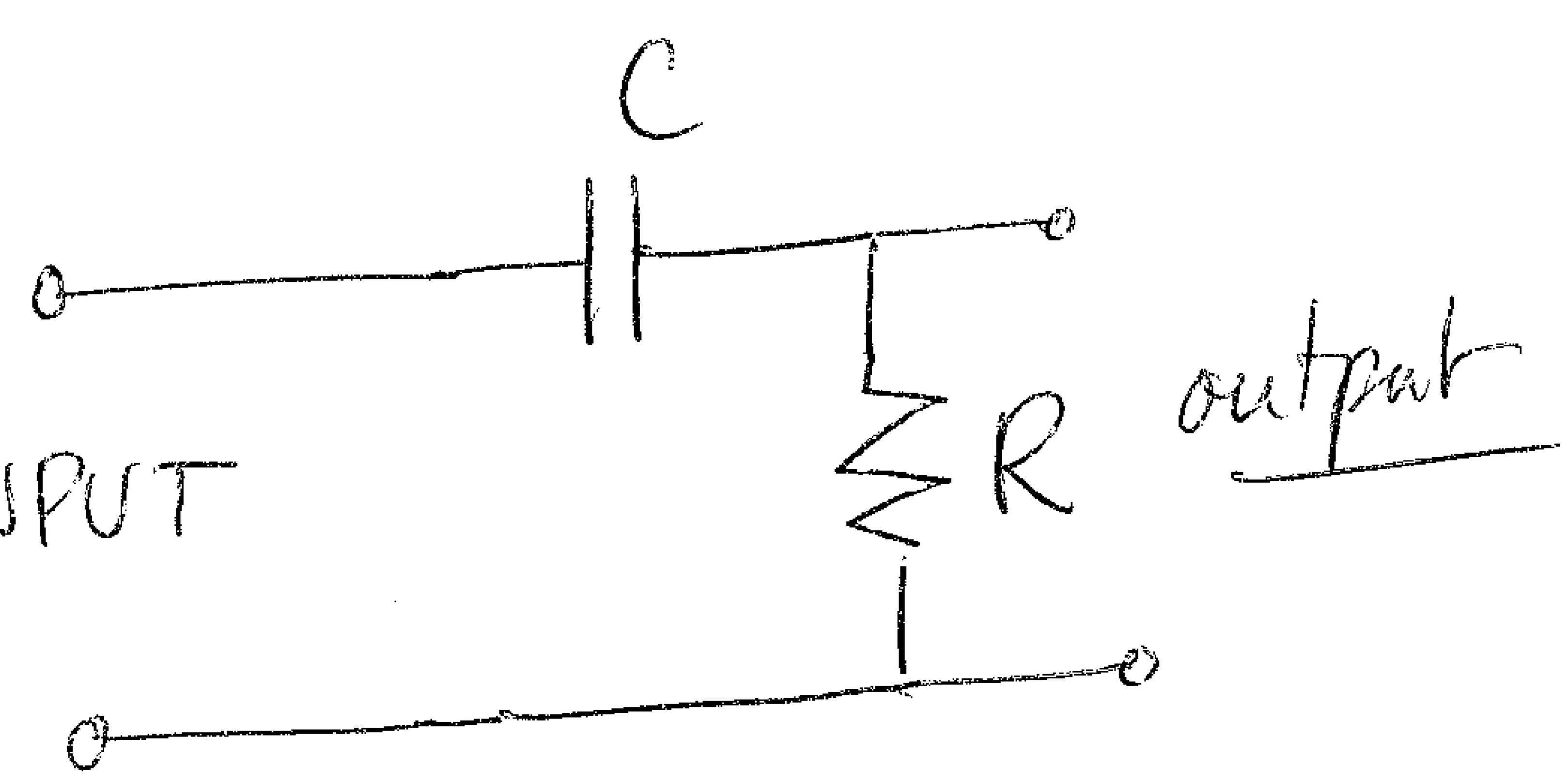
Q14. Express the complex number **z = - 6.00 + *i*(4.00)** in exponential form:

$|z| = \sqrt{(-6)^2 + 4^2} = \sqrt{52} = 7.21$

$\theta = \tan^{-1}\left(\frac{4}{-6}\right) + \pi = -0.588 + 3.142 = 2.554 \text{ rad}$

Q14. $\sqrt{52}\, e^{i(2.55)}$

Q15. Draw and label a simple circuit that can be used as a high-pass filter.



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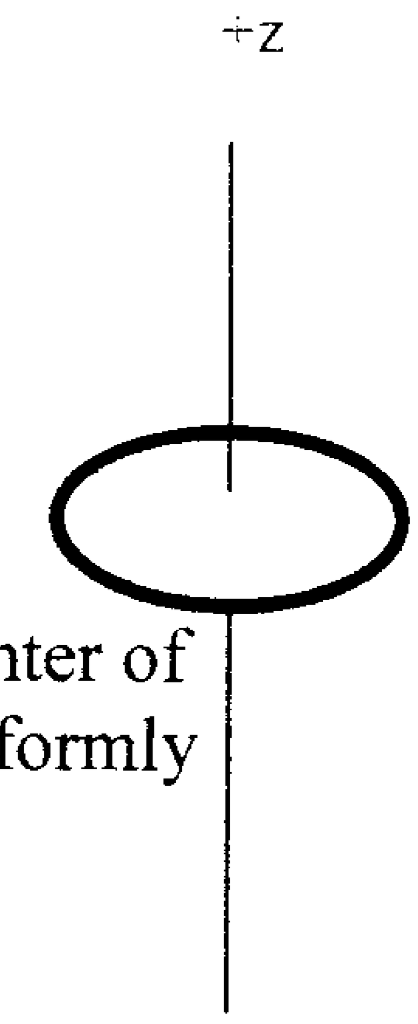


Problem 1. (20 points) Show your work. Partial credit will depend on that.

Consider a circular ring of positive charge. The ring lies in the xy plane and the center of the ring is at the origin. The radius of the ring is b, and the total charge Q > 0 is uniformly distributed on the ring. Express your answers below in terms of Q, b, and physical constants.

5 (A) Determine λ, the charge per unit length on the ring.

$$\lambda = \frac{Q}{2\pi b}$$

5 3 2 (B) Using the usual convention that the electrostatic potential is zero at great (infinite) distance from the origin, what is the electrostatic potential V_{origin} at the center of the ring? Justify your answer.

Since all of the charge is at the same distance (b) from the origin, $V_{origin} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{dq_i}{r_i} = \frac{Q}{4\pi\epsilon_0 b}$

5 3 2 (C) Determine the electrostatic potential as a function of z along the z axis: V(0,0,z) = ? Justify your answer.

Again, all parts of the ring are at the same distance ($\sqrt{b^2+z^2}$) from the point (0,0,z).

$$\therefore V(0,0,z) = \frac{Q}{4\pi\epsilon_0\sqrt{b^2+z^2}}$$

(Problem 1 continues on the next page.)

5)

Problem 1. (continued)

(D) Use your answer to part (C) to determine the electric field **E**(**r**) for $\vec{r} = 0\hat{i} + 0\hat{j} + 3b\hat{k}$. Show your work and be sure to express the electric field as a vector.

$$\vec{E} = -\vec{\nabla} V$$

By symmetry, $\vec{E}$ has only a z component (as long as we stay on the z axis).

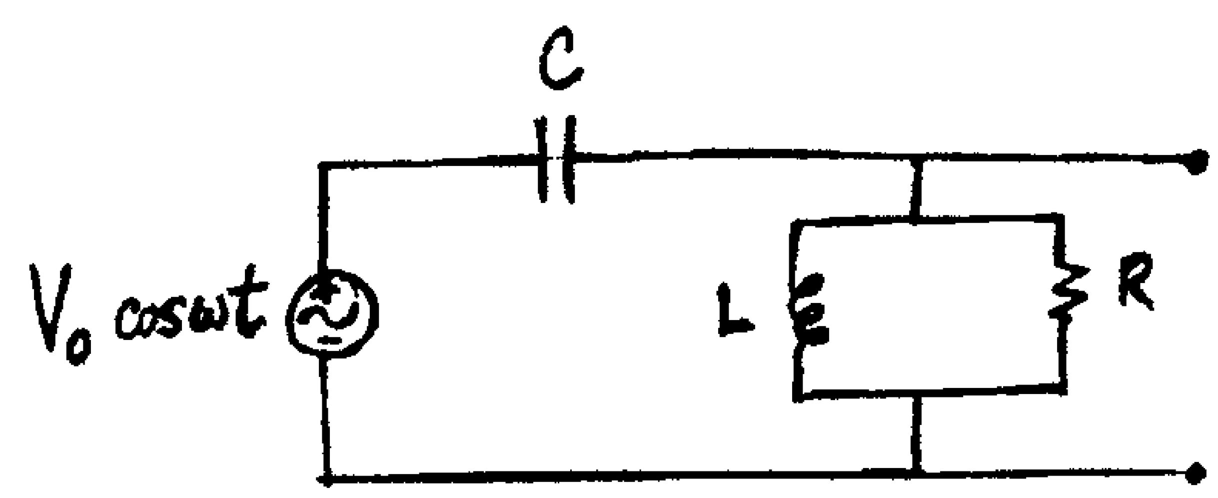
$$\text{So} \quad \vec{E} = -\hat{k}\frac{\partial V}{\partial z} = -\hat{k}\left(\frac{Q}{4\pi\epsilon_0}\right)\frac{\partial}{\partial z}\left((b^2+z^2)^{-1/2}\right)$$

$$= -\hat{k}\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{2}\frac{(2z)}{(b^2+z^2)^{3/2}}\right] = \hat{k}\frac{Q}{4\pi\epsilon_0}\frac{z}{\left(\sqrt{b^2+z^2}\right)^3}$$

For $z = 3b$, get

$$\underline{\vec{E}} = \hat{k}\frac{Q}{4\pi\epsilon_0}\frac{3b}{(10b^2)^{3/2}} = \underline{\hat{k}\frac{3}{10\sqrt{10}}\left(\frac{Q}{4\pi\epsilon_0 b^2}\right)}$$

Problem 2. (20 points) Show your work. Partial credit will depend on that.



Consider the electric circuit shown above. Note that this is NOT a series RLC circuit! Give your answers for (A)-(C) below in terms of V_0, ω, C, L, and R and any necessary mathematical constants.

5

(A) What is the (complex) impedance Z_C of the capacitor? (Express it in rectangular form.)

$$Z_c = -iX_c = \frac{-i}{\omega C} = 0 - i\left(\frac{1}{\omega C}\right)$$

5

(B) Determine the (complex) impedance Z_2 of the parallel LR combination. (Express your result in rectangular form.)

$$Z_2 = \frac{Z_L Z_R}{Z_L + Z_R} = \frac{(i\omega L)R}{i\omega L + R}$$

$$Z_2 = \frac{i\omega L R(R - i\omega L)}{R^2 + (\omega L)^2} = \frac{R(\omega L)^2}{R^2+(\omega L)^2} + \frac{i(\omega L)R^2}{R^2+(\omega L)^2}$$

5

(C) Show that the (complex) impedance Z_{final} of the series combination of the capacitor and the LR (parallel) combination is given by $Z_{final} = \frac{(\omega L)^2 R}{R^2 + (\omega L)^2} + i\left[\frac{(\omega L)R^2}{R^2 + (\omega L)^2} - \frac{1}{\omega C}\right]$

Series combination $\Rightarrow$

$$Z_{final} = Z_c + Z_2 = \frac{(\omega L)^2 R}{R^2+(\omega L)^2} + i\left[\frac{(\omega L)R^2}{R^2+(\omega L)^2} - \frac{1}{\omega C}\right]$$

(Problem 2 continues on the next page.)

Problem 2. (continued)

5)

(D) If the sinusoidal voltage source is represented by the phasor $\mathbf{V}_0 = V_o e^{i\omega t}$ and the (rightward) current through the capacitor is represented by the phasor $\mathbf{I}_0 = I_o e^{i(\omega t+\phi)}$, <u>describe how to determine</u> the magnitude I_o and phase constant ϕ of the current phasor. You do not need to actually determine their values.

$$\mathbb{V}_0 = V_0 e^{i\omega t} = \mathbb{I}_0 Z_{final}, \quad \text{so}$$

$$I_0 e^{i(\omega t+\phi)} = \frac{V_0 e^{i\omega t}}{Z_{final}}$$

That is, find the current phasor $\mathbb{I}_0$ by dividing the voltage phasor $\mathbb{V}_0$ by the complex impedance Z_{final}.

Obtaining I_0 + ϕ will be easiest ~~&~~ if Z_{final} is re-expressed in exponential form.