Physics 260
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Name Key
September 27, 2002

HOUR TEST #1

HONOR PLEDGE:

In completing this test, I have neither given nor received unauthorized assistance.

_____________________
(signature)

- This is a closed-book, closed-notes test. No written material is allowed.
- You are permitted to use an electronic calculator only to assist with numerical work.
- Assume that the given quantities are accurate enough to justify three (and only three) figures in your final answers.
- **Show your work, especially on the problems!** Credit, especially partial credit on problems where your final answer is incorrect, will depend on the work shown.

**The test consists of 15 questions and 2 problems. Perfect score is 100.**
**Questions 1-15 count 4 points each. (60%)**
**Problems 1-2 count 20 points each. (40%)**

Possibly useful information.

terrestrial acceleration due to gravity: $g = 9.80\ m/s^2$
$\pi = 3.14159265$ $e = 2.71828183$ $\log_{10}e = 0.43429448$
$1\ eV = 1.602 \times 10^{-19}\ J$ 1 year $= 3.156 \times 10^7$ sec
astronomical unit (AU) $= 1.496 \times 10^{11}$ m
speed of light in vacuum: $c = 2.998 \times 10^8\ m/s$
constant of gravitation: $G = 6.673 \times 10^{-11}\ N\text{-}m^2/kg^2$
Planck's constant: $h = 6.626 \times 10^{-34}\ J\text{-}s = 4.136 \times 10^{-15}\ eV\text{-}s$
"h-bar" $\hbar \equiv h/2\pi = 1.055 \times 10^{-34}\ J\text{-}s = 6.583 \times 10^{-16}\ eV\text{-}s$
Boltzmann constant: $k = 1.381 \times 10^{-23}\ J/K = 8.617 \times 10^{-5}\ eV/K$
Avogadro's number $N_A = 6.022 \times 10^{23}$ particles/mole
Universal gas constant: $R = N_A k = 8.315\ J/(mol\text{-}K)$
Stefan-Boltzmann constant: $\sigma = 5.671 \times 10^{-8}\ W/(m^2K^4)$
unified mass unit (dalton): $(1\ u) = 1.661 \times 10^{-27}\ kg$
energy equivalent of 1 dalton: $(1u)c^2 = 1.493 \times 10^{-10}\ J = 931.9\ MeV$
proton (rest) mass: $m_p = 1.673 \times 10^{-27}\ kg$
rest energy of proton: $m_pc^2 = 1.504 \times 10^{-10}\ J = 938.6\ MeV$
electron (rest) mass: $m_e = 9.109 \times 10^{-31}\ kg$
rest energy of electron: $m_ec^2 = 9.187 \times 10^{-14}\ J = 0.5111\ MeV$
elementary unit of charge: $e = 1.602 \times 10^{-19}\ C$
product of h and c: $hc = 1.987 \times 10^{-25}\ J\text{-}m = 1.240 \times 10^{-6}\ eV\text{-}m = 1240\ eV\text{-}nm$
standard temperature: $T = 0\ °C = 273.15\ K$
standard pressure: $P = 1.000$ atmosphere $= 1.013 \times 10^5\ N/m^2$

Questions 1-15. (4 points each) On many of these questions, partial credit is not available. You do not need to show your work.

Q1. What is the value of the terrestrial acceleration due to gravity at a distance of 5.00 Earth radii from the center of the Earth? (Ignore all gravitating bodies other than the Earth.) Give your answer in SI units.

$$g(r) = g(R_\oplus)\left(\frac{R_\oplus}{r}\right)^2$$

Q1. $g(r=5R_\oplus) = \frac{9.80\,m/s^2}{25} = 0.392\,m/s^2$

Q2. The semi-major axis of Earth's orbit is 1.00 AU. The semi-major axis of Neptune's orbit is 30.1 AU. How many Earth-years does it take for Neptune to complete one orbit of the Sun?

$$T^2(yr^2) = \frac{1\,yr^2}{AU^3}\,a^3(AU) \qquad T(yr) = \sqrt[2]{(30.1)^3}$$

Q2. 165 (yr)

Q3. At what point in its orbit is the Earth's orbital speed the greatest?

(A) aphelion
(B) autumnal equinox
(C) perihelion
(D) summer solstice
(E) vernal (spring) equinox
(F) winter solstice
(G) All of the above: the Earth's orbital speed is a constant.

Q3. (C) perihelion

Q4. What is the defining characteristic of a conservative force?

That the work done by the force between any two points ($P_i$ and $P_f$) is INDEPENDENT of the path. [OR The work done about any closed path is zero.]

Q5. What is the defining characteristic of a rigid body?

A rigid body is an object within which the distance between any two particles of the body remains constant as the body moves.

Q6. The ________________________ at any point P is defined as the ratio of the net gravitational force to the mass of a test particle placed at P.

- (A) gravitational field
- (B) gravitational flux
- (C) gravitational potential
- (D) gravitational potential energy
- (E) None of the above answers is correct.

Q6. (A) gravitational field

Q7. In fluid mechanics, the equation that directly expresses the conservation of matter is ______________.

- (A) Bernoulli's equation
- (B) the equation of continuity
- (C) Pascal's law
- (D) the barometric equation
- (E) None of the above answers is correct.

Q7. (B) the equation of continuity

Q8. What is the defining characteristic of a stable equilibrium?

If the ~~to~~ system is displaced slightly from the equilibrium, forces (and/or torques) arise that ~~tend to~~ move the system back toward the equilibrium.

Q9. Kepler's 2nd Law (law of equal areas) is equivalent to the statement that a planet's __________________ per unit mass remains constant as the planet orbits.

- (A) orbital kinetic energy
- (B) orbital potential energy
- (C) total energy
- (D) orbital angular momentum
- (E) total linear momentum

Q9. (D) orbital angular momentum

Q10. The SI unit of stress is ____________.

Q10. $N/m^2$ or Pascals

Q11. Describe a situation in which the classical work-energy (CWE) "theorem" fails. You needn't explain the failure in detail – just give one specific example.

The motion of a bicycle which is being pedaled from rest. (~~under~~ Static friction accelerates the bicycle but does no work.)

Q12. If a force has an associated potential energy, the force at any location always points ________________.

(A) in the direction in which the potential energy <u>increases</u> most rapidly with distance
(B) in a direction in which the potential <u>does not vary</u> with distance
(C) in the direction in which the potential energy <u>decreases</u> most rapidly with distance
(D) None of the above answers is correct: There's no general relationship between the direction of the force and the spatial variation of the associated potential energy.

Q12. C

Q13. A particle of mass 3.00 kg has instantaneous position $\vec{r} = (4.00m)\hat{i}$ and instantaneous velocity $\vec{v} = (2.00m/s)\hat{i} + (5.00m/s)\hat{j}$. What is the magnitude of its instantaneous orbital angular momentum about the origin?

(A) 24.0 kg-m/s
(B) 60.0 kg-m/s
(C) 64.6 kg-m/s
(D) 84.0 kg-m/s
(E) This can't be determined without more information.

Q13. (B) 60.0 kg·m/s

Q14. A wheel of radius 0.200 m is rolling without slipping along the floor. It is observed to complete 3.00 rotations each second. What is the translational speed of the center of the wheel?

$v = \omega R = (6\pi \text{ rad/sec})(0.200 \text{ m})$

(A) $6.67 \times 10^{-2}$ m/s
(B) 0.600 m/s
(C) 3.77 m/s
(D) This can't be determined without more information.

Q14. (C) 3.77 m/s

Q15. When a 100 kg object is hung from an elastic cable, the cable is observed to stretch by 6.00 millimeters. If the same object were suspended from 4 such cables arranged side by side (to share the load equally), how much would each one of the cables stretch?

$mg = F_1 = k\Delta\ell_1$

$mg = 4F' = 4(k\Delta\ell')$

$\Rightarrow 4\Delta\ell' = \Delta\ell_1$

Q15. 1.50 millimeters

**Problem 1.** **(20 points) Show your work. Partial credit will depend on that.**

The planet Gorgon has mass M and radius R. (Assume that it is not rotating, that it has a spherically symmetric mass distribution, and that its atmosphere is so thin that it can be neglected.) Express all your answers in terms of G, M, R. and other given quantities.

(A)-(B) A visiting spaceship (commanded by Captain Goofball) is hovering at a distance 3R from the center of the planet, and it releases a probe (actually a waste canister!) of mass $m_w$ which free-falls toward the planet's surface. (Neglect the gravitational attraction between the spaceship and the canister.)

(A) Find the kinetic energy of the canister just before it hits the surface.

$$\frac{1}{2} m_w v_i^2 - \frac{GMm_w}{r_i} = \frac{1}{2} m_w v_f^2 - \frac{GMm_w}{r_f}$$

$$v_i = 0,\ r_i = 3R,\ r_f = R \Rightarrow$$

$$\frac{1}{2} m_w v_f^2 = \frac{GMm_w}{R} - \frac{GMm_w}{3R} = \frac{2GMm_w}{3R}$$

(B) Find the speed of the canister just before it hits the surface.

$$v_f = \sqrt{\frac{4GM}{3R}}$$

(Problem 1 continues on the next page.)

**Problem 1.** ***(continued)***

(C)-(D) The hovering spaceship fires another canister of the same mass $m_w$ <u>forward</u> into a circular orbit of radius 3R around the planet.

(C) What is that canister's kinetic energy? need $-\frac{m_w V_c^2}{(3R)} = -\frac{GMm_w}{(3R)^2}$

$$\Rightarrow \quad V_c = \sqrt{\frac{GM}{3R}}$$

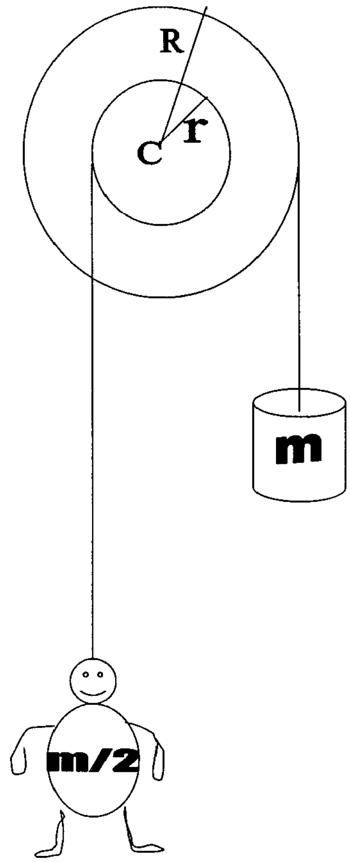
(D) Assuming that the spaceship has fired rockets to prevent recoil from the launch of the second canister and is still hovering over Gorgon, find out how much time Captain Goofball has available to get out of the way before the orbiting canister crashes into the spaceship.

time available = one orbit

$$= \frac{2\pi(3R)}{V_c} = \underline{\underline{2\pi\sqrt{\frac{(3R)^3}{GM}}}}$$

$$= \underline{\underline{\frac{2\pi(3R)^{3/2}}{(GM)^{1/2}}}}$$

**Problem 2. (20 points) Show your work. Partial credit will depend on that.**




In the figure at the right, the two disks (of one radius r, the other of radius R) are joined together and they freely rotate as a unit about the fixed horizontal axis C. The disk assembly by itself has rotational inertia $I_D$ about the symmetry axis C. There are two cords. The cord holding the canister of mass m is wrapped around the outside rim and unwinds if the system moves clockwise. The cord holding the gumby (which has mass m/2) is wrapped around the inner rim (radius r) and winds up if the system turns clockwise. The system is released from rest, and the canister descends with constant acceleration.

**The radius of the inner rim r = R/2.**

(A) What is the rotational inertia of the whole system with respect to the fixed axis C?

$$I = I_D + m_{gumby} r^2 + mR^2$$

$$= I_D + \frac{m}{2}\left(\frac{R}{2}\right)^2 + mR^2$$

$$I = I_D + \frac{9mR^2}{8}$$

(B) Find the net external torque about the axis C exerted on the system by Earth's gravity?

torque on disk itself $= 0$

torque ~~on~~ due to ~~gr~~ weight of canister $= mgR$ (cw)

torque due to weight of gumby $= \frac{m}{2} g \frac{R}{2}$ (ccw)

Net external torque $= \frac{3mgR}{4}$ clockwise

*Problem 2 continues on the next page.*

$\left(-\frac{3mgR}{4} \text{ if use CCW} = +\right)$

**Problem 2.** *(continued)*

$\circlearrowleft +$

(C) Find the angular acceleration of the disk assembly

$$I\alpha = \tau_{net}$$

$$\alpha = \frac{-(3mgR/4)}{\left(I_D + \frac{9mR^2}{8}\right)}$$

At the instant when the canister has descended a distance h, . .

(D) . . . how far has the gumby ascended?

a distance $\frac{h}{2}$, since

$$v_{gumby} = \omega r = \omega \frac{R}{2} = \frac{1}{2}\left(\text{speed of canister}\right)$$

(E) . . . what is the total kinetic energy of the system? (HINT: The CWE theorem can be used here.)

$$KE + PE = \text{constant}$$

$$KE_f + PE_f = KE_i + PE_i$$

$$KE_f = KE_i + (PE_i - PE_f)$$

$$KE_f = 0 + mgh - \frac{m}{2}g\frac{h}{2}$$

$$= \frac{3mgh}{4}$$

NOTE: $mgh = (PE_i - PE_f)_{canister}$

$-\frac{m}{2}g\frac{h}{2} = (PE_i - PE_f)_{gumby}$