Physics 260
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Name Key
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HOUR TEST #2

HONOR PLEDGE:

In completing this test, I have neither given nor received unauthorized assistance.

(signature)

- This is a closed-book, closed-notes test. No written material is allowed.
- You are permitted to use an electronic calculator only to assist with numerical work.
- Assume that the given quantities are accurate enough to justify three (and only three) figures in your final answers.
- **Show your work, especially on the problems!** Credit, especially partial credit on problems where your final answer is incorrect, will depend on the work shown.

The test consists of 15 questions and 2 problems. Perfect score is 100.
Questions 1-15 count 4 points each. (60%)
Problems 1-2 count 20 points each. (40%)

Possibly useful information.

terrestrial acceleration due to gravity: $g = 9.80\ m/s^2$
$\pi = 3.14159265$ $\quad$ $e = 2.71828183$ $\quad$ $\log_{10}e = 0.43429448$
$1\ eV = 1.602 \times 10^{-19}\ J$ $\quad$ $1\ year = 3.156 \times 10^7\ sec$
astronomical unit (AU) = 1.496×10^{11} m
speed of light in vacuum: $c = 2.998 \times 10^8\ m/s$
constant of gravitation: $G = 6.673 \times 10^{-11}\ N\text{-}m^2/kg^2$
Planck's constant: $h = 6.626 \times 10^{-34}\ J\text{-}s = 4.136 \times 10^{-15}\ eV\text{-}s$
"h-bar" $\hbar \equiv h/2\pi = 1.055 \times 10^{-34}\ J\text{-}s = 6.583 \times 10^{-16}\ eV\text{-}s$
Boltzmann constant: $k = 1.381 \times 10^{-23}\ J/K = 8.617 \times 10^{-5}\ eV/K$
Avogadro's number $N_A = 6.022 \times 10^{23}$ particles/mole
Universal gas constant: $R = N_A k = 8.315\ J/(mol\text{-}K)$
Stefan-Boltzmann constant: $\sigma = 5.671 \times 10^{-8}\ W/(m^2K^4)$
unified mass unit (dalton): $(1\ u) = 1.661 \times 10^{-27}\ kg$
energy equivalent of 1 dalton; $(1u)c^2 = 1.493 \times 10^{-10}\ J = 931.9\ MeV$
proton (rest) mass: $m_p = 1.673 \times 10^{-27}\ kg$
rest energy of proton: $m_pc^2 = 1.504 \times 10^{-10}\ J = 938.6\ MeV$
electron (rest) mass: $m_e = 9.109 \times 10^{-31}\ kg$
rest energy of electron: $m_ec^2 = 9.187 \times 10^{-14}\ J = 0.5111\ MeV$
elementary unit of charge: $e = 1.602 \times 10^{-19}\ C$
product of h and c: $hc = 1.987 \times 10^{-25}\ J\text{-}m = 1.240 \times 10^{-6}\ eV\text{-}m = 1240\ eV\text{-}nm$
standard temperature: $T = 0\ ^\circ C = 273.15\ K$
standard pressure: $P = 1.000$ atmosphere $= 1.013 \times 10^5\ N/m^2$

Questions 1-15. (4 points each) On most of these questions, partial credit is not available. You are not required to show your work.

Q1-3. Consider a traveling pressure wave described by the following equation:

$$\Psi(x,t) = (1.20\times10^{-2}\,Pa)\cos[(5.00 rad/m)x - (1.80\times10^{3} rad/s)t + 0.300 rad]$$

Q1. This wave is traveling in the ________________.

(A) +x direction
(B) –x direction
(C) The direction of travel cannot be determined without additional information.

Q1. A

Q2. What is the speed of this wave? (Include correct SI units.)

$$V = \frac{\omega}{k} = \frac{1800 \text{ rad/s}}{5.00 \text{ rad/m}}$$

Q2. 360 m/s

Q3. What is the period of this wave? (Include correct SI units.)

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{1800 \text{ rad/s}}$$

Q3. 3.49×10^{-3} s

Q4. Two sinusoidal waves are interfering to produce 5 beats per second. One of the sinusoidal waves has a frequency of 440 Hz. What are the two possible frequencies for the other sinusoidal wave?

440 ± 5

Q4. 435 Hz **OR** 445 Hz

Q5. At T =273 K, a metal bar has a length of 2.00 m. When the bar is cooled to T = 223 K, its length shrinks by 1.50 x 10^{-3} m. Determine the coefficient of thermal expansion for this metal, including appropriate SI units.

$$\alpha = \frac{1}{l}\frac{\Delta l}{\Delta T} = \frac{1}{2.00\text{m}}\frac{(-1.50\times10^{-3}\text{m})}{(-50\text{ K})}$$

Q5. 1.50×10^{-5} K^{-1}

Q6. What is the defining characteristic of a thermal reservoir?

It is a system with such a large heat capacity that its temperature is unaffected by any heat transfer in or out of the reservoir.

Q7. In the equation $\frac{\delta Q}{dt} = -kA\frac{dT}{dx}$, what are the correct SI units for the quantity k?

$\frac{\text{Joules}}{\text{sec}} = [k](m^2)\left(\frac{K}{m}\right) \rightarrow [k] = \frac{\text{watts}}{m \cdot K}$

Q7. $\frac{\text{watts}}{m \cdot K}$ or $\frac{\text{Joules}}{m \cdot s \cdot K}$

Q8. In the quasi-static adiabatic expansion of an ideal gas, the entropy of the gas ___________.

(A) increases
(B) decreases
(C) remains the same
(D) This cannot be determined without additional information.

Q8. C

Q9. Both helium (atomic mass 4.00) and argon (atomic mass 39.9) are monatomic gases. In a gas which is a mixture of helium and argon, what is the ratio of the average speed of the helium atoms to the average speed of the argon atoms?

(A) 0.0101
(B) 0.100
(C) 0.316
(D) 3.16
(E) 9.98
(F) 99.5
(G) It cannot be determined without knowing the temperature of the mixture.

$\frac{\overline{v}_{He}}{\overline{v}_{Ar}} = \sqrt{\frac{m_{Ar}}{m_{He}}} = 3.16$

Q9. ~~3.16~~ D

Q10. In the simplest model of a monatomic solid, there are 6 active degrees of freedom per atom. On the basis of this model, the predicted molar specific heat (at constant volume) of the solid is _______________.

(A) 3R
(B) 6R
(C) 9R
(D) 12R

6 d.o.f. $\times \left(\frac{1}{2}R \text{ for each one}\right)$

3 kinetic + 3 pot'l

Q10. A

Q11. In a typical diatomic gas at room temperature, the adiabatic exponent $\gamma = 7/5 = 1.40$, because at this temperature there are 5 active degrees of freedom. Indicate how many of these 5 active degrees of freedom are of each of the types below:

Q11. translational: 3

rotational: 2

vibrational: 0

Q12. Define in words the efficiency of a heat engine (not necessarily a Carnot engine).

This is the ratio of work output to high-temperature heat input.

Q13. (TRUE OR FALSE) If a gas is taken quasi-statically around any closed path (cycle) in the PV plane, the integral of ratio of heat added to absolute temperature equals zero: $\oint \frac{\delta Q}{T} = 0$.

Q13. TRUE

Q14. Identify each of the following variables as <u>intensive</u> (independent of size) OR <u>extensive</u> (proportional to size):

Q14.
pressure intensive
entropy extensive
temperature intensive
density intensive

Q15. (TRUE OR FALSE) In thermodynamic equilibrium, all <u>macrostates</u> of a system are equally likely.

Q15. FALSE

(all micro states are equally likely
NOT all macrostates)

Problem 1. (20 points) Show your work. Partial credit will depend on that.

A cyclic heat engine that operates between a highest temperature of 800 K and a lowest temperature of 300 K has a work output of 5.40 megajoules (MJ) per cycle and rejects waste (low-temperature) heat of 7.20 MJ per cycle.

(A) If the power output (work output per unit time) of this heat engine is 400 kilowatts (kW), what is the duration of one cycle?

$$(400\ \text{kW}) \times \Delta t = W = 5.40 \times 10^3\ \text{kJ}$$

$$\rightarrow \underline{\Delta t} = \frac{5400\ \text{kJ}}{400\ \text{kW}} = \underline{13.5\ \text{sec}}$$

(B) What is the heat input per cycle?

$$\underline{Q_H} = W + |Q_c| = 5.40\ \text{MJ} + 7.20\ \text{MJ} = \underline{12.6\ \text{MJ}}$$

(C) What is the efficiency of this heat engine?

$$\underline{\varepsilon} = \frac{W}{Q_H} = \frac{5.40\ \text{MJ}}{12.6\ \text{MJ}} = \underline{0.429} = \underline{42.9\%}$$

(D) If a Carnot engine could be used between a hot reservoir of 800 K and a cold reservoir at 300 K, how much work output per cycle could it achieve given the same heat input found in part B?

$$\underline{W_{carnot}} = \varepsilon_{carnot} Q_H = \left(\frac{T_H - T_c}{T_H}\right)(12.6\ \text{MJ})$$

$$= \left(1 - \frac{300}{800}\right)(12.6\ \text{MJ}) \approx \underline{7.88\ \text{MJ}}$$

Problem 2. (20 points) Show your work. Partial credit will depend on that.

Neon gas (which is monatomic) is confined at a pressure $P_o = 1.00 \times 10^4$ pascals and a temperature $T_o = 800$ K. The volume occupied by the gas is $V_o = 1.00\ m^3$.

(A) Find the number of moles of neon gas.

$$PV = nRT \Rightarrow n = \frac{PV}{RT} = \frac{(1.00 \times 10^4 \text{ Pascals})(1.00\ m^3)}{\left(8.315 \frac{J}{mol \cdot K}\right)(800\ K)}$$

$$n = 1.50 \text{ moles}$$

(B)-(C) This gas undergoes an isobaric compression to volume $V_1 = 0.500V_o$.

(B) What is the gas temperature T_1 at the completion of this isobaric compression?

$$\frac{P}{nR} = cst \Rightarrow \frac{T}{V} = cst \Rightarrow \frac{T_1}{V_1} = \frac{T_0}{V_0} \Rightarrow T_1 = T_0\left(\frac{V_1}{V_0}\right)$$

$$T_1 = 400\ K$$

(C) During the isobaric compression, does the entropy of the neon gas increase, decrease, or remain the same?

it decreases Explain your choice. both V & T decrease ⇒ both the spatial disorder + the disorder of motion decrease

Problem 2 continues on the next page.

Problem 2. (continued)

(D)-(E) Following the isobaric compression described above, the gas is allowed to expand quasi-statically and adiabatically from V_1 to $V_2 = 2.000V_1 = V_o$.

(D) What is the gas temperature T_2 at the completion of the adiabatic expansion? $\gamma = 5/3$

$$PV^{\gamma} = cst \Rightarrow TV^{\gamma-1} = cst'$$

$$T_1 V_1^{2/3} = T_2 V_2^{2/3}$$

$$\Rightarrow \underline{\underline{T_2}} = T_1 \left(\frac{V_1}{V_2}\right)^{2/3} = (400\,K)(0.500)^{2/3} = \underline{\underline{252\,K}}$$

(E) How much work is done by the gas during the adiabatic expansion?

Can use $W_{by} = \int P\,dV$

OR more simply can use

$$\Delta U_{\substack{ad\\exp}} = Q_{\substack{ad\\exp}} - W_{\substack{by\\ad\,exp}}$$

$$Q_{\substack{ad\\exp}} = 0$$

$$\Rightarrow W_{\substack{by\\ad\,exp}} = -\Delta U_{\substack{ad\\exp}} = -nC_V(T_2 - T_1)$$

$$= -(1.50)\left(\tfrac{3}{2}\right)\left(8.315\,\frac{J}{mole\cdot K}\right)(252-400)K$$

$$\underline{\underline{W_{\substack{by\\ad\,exp}}}} = 2.769\times10^3\,J \simeq \underline{\underline{2.77\,kJ}}$$