Reese Ch. 27, Part B (pp. 1257-1261)

- Requirements for Theory of Particle-Waves
- Characteristics of the Wavefunction
- The Loss of Classical Determinism

Reminder: Observation and Measurement

- Within classical physics, it is assumed possible when observing a system to either make the disturbance negligible or else to account for it precisely.
- In the domain of quantum physics (where the Heisenberg uncertainty products can't be ignored), the very act of measurement changes the system in ways that cannot be accounted for precisely. In general, the predictions of quantum mechanics have to be expressed in statistical (probabilistic) terms.

Requirements for a Theory of Particle-Waves

- The wavefunction should be "large" where the particle is likely to be detected and "small" where the particle is not likely to be detected.
- The theory must incorporate the de Broglie relation:

$$\boxed{\lambda = \frac{h}{p} \Leftrightarrow p = \hbar k}$$

- Theory must incorporate the fact that low-flux ("one particle-at-a-time") experiments reveal the same diffraction pattern as high-flux experiments.

Characteristics of the Wavefunction Ψ: I

- Amplitude of Ψ must determine likelihood of particle detection.
- Wavefunction Ψ must be able to interfere with itself in order to yield the observed diffraction pattern.
- Wavefunction Ψ must represent the behavior of individual particles because even low-flux experiments yield the diffraction pattern.
- It would be "nice" if we could use the wavefunction Ψ to calculate the results of any experiment we choose to perform. (We must accept the fact that, in general, the predictions will be statistical ones.)

Characteristics of the Wavefunction Ψ: II

- Probabilities are inherently non-negative; to get the required destructive interference, the wavefunction Ψ must not be restricted to non-negative numbers.
- We are accustomed to thinking of waves of definite wavelength as real sinusoids, but a definite-momentum wavefunction Ψ can't be a real sinusoid if it is to represent a particle that is equally likely to be found anywhere along the "beam path."
- We can satisfy these two requirements by using a complex exponential for a definite-momentum wave function. . .

Characteristics of the Wavefunction Ψ: III

- The wavefunction for a particle of definite momentum $p = \hbar k$ will be written as:

$$\boxed{\Psi(x,t) = Ae^{i(kx-\omega t)} = A\left[\cos(kx-\omega t) + i\sin(kx-\omega t)\right]}$$

- The likelihood of finding the particle at any given location is given by $|\Psi|^2 \equiv \Psi^*\Psi$:

$$\boxed{\Psi^*\Psi = A^* e^{-i(kx-\omega t)} \times Ae^{i(kx-\omega t)} = A^*A = |A|^2}$$

- A particle of definite momentum is equally likely to be found anywhere along the beam.

Characteristics of the Wavefunction Ψ: IV

- Since it is complex-valued, the wavefunction is not directly observable. Rather, it is a function (THE function) that we can use to predict numbers (necessarily real) which can be checked experimentally.
- The wavefunction Ψ is also referred as the **probability amplitude**, because its squared magnitude is the probability per unit distance Δx.

Characteristics of the Wavefunction Ψ: V

- By analogy with the physics of classical waves, we expect that the wavefunction Ψ satisfies a **linear wave equation**.
- This equation (to be developed later) is nonrelativistic: it applies to a particle of nonzero mass m and kinetic energy much less than mc^2.
- One clue about this equation comes from de Broglie's analysis. If we consider a free particle and assign it zero potential energy, then $E = p^2/2m$. Then

$$\boxed{E = \hbar\omega \text{ plus } p = \hbar k \text{ implies } \hbar\omega = \frac{\hbar^2 k^2}{2m}}$$

The Loss of Classical Determinism

- Newtonian physics was developed to analyze the behavior of macroscopic objects. In retrospect, the fact that its categories fail to capture the details of behavior at the microscopic level should not be surprising.
- It is startling to realize that we can neither neglect "the disturbance caused by observation" nor take it into account and thereby preserve strict determinism.

  (Thankfully, quantum mechanics does still maintain causality in a limited sense.)
- Of course, in a practical sense, Newtonian physics remains a fine, fine theory of behavior as long as our tolerances are much coarser than the limits imposed by the uncertainty principles.