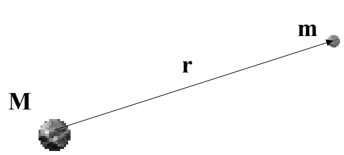
Reese Chapter 6, part A

- Newton's Law of Gravitation
- Attraction of Spherical Distributions
- Earth's Mass
- Artificial Satellites

Newtonian Gravitation

- Evidence for the form of Newton's gravitational force between mass points
  - Dependence on separation (distance):
    - Observed acceleration of Moon vs. gravitational acceleration of objects at Earth's surface
  - Dependence on masses:
    - Newton's third law $\rightarrow$ symmetry between roles of M & m
    - Newton's second law *plus* common gravitational acceleration of objects near Earth's surface $\rightarrow$ the gravitational force is proportional to product Mm

Mathematical Form of the Law



$$\left|\vec{F}\right| = \frac{GMm}{r^2}$$

Universality of Gravitation

- A force of attraction between *each* pair of point particles in the universe
- G, the gravitational constant, is usually assumed to be a constant of nature that must be determined from observation or experiment (e.g., Cavendish, 1798)

The overall force of distributed matter on a point mass: I

- General rule: Apply the principle of superposition of forces: construct the vector sum of the individual attractions from all the parts of the distribution.

The overall force of distributed matter on a point mass: II

- Results for uniform (i.e, spherically symmetric) thin shell (Appendix A of text)
  - If the point mass is anywhere outside the shell, then the force on the point mass is exactly what it would be if the shell's entire mass were concentrated at its center.
  - If the point mass is anywhere inside the shell, then the individual attractions precisely cancel, so there is zero overall force on the point mass.

The overall force of distributed matter on a point particle: III

- Results for a spherically symmetric mass distribution follow by the superposition principle from the results for a thin spherical shell:
  - If the particle is anywhere outside the sphere, then the force on the particle is exactly what it would be if the entire attracting mass were concentrated at the center of the sphere.
  - If the particle is anywhere inside the sphere, the "surrounding" matter exerts zero force, so the force is obtained by applying the preceding rule. The only mass that counts is the "interior" mass.

Determining the Total Mass of the Earth

- The gravitational acceleration of any object at Earth's surface can be measured:

$$g \approx 9.80 \text{ m/s}^2$$

- Object's weight is just the Earth's attraction for it , so $W = GMm/R^2$
- But $g = W/m$, so $g = GM/R^2 \rightarrow$

$$M = gR^2/G$$

- Questions: What assumptions and approximations have been made? What's been neglected?

Artificial Satellites of the Earth

- Anticipated by Newton in 1687 (270 years before Sputnik I).
- An Earth satellite in a circular orbit of radius r has a steady speed $v = 2\pi r/T$, where T is the period. Because the Earth's provides the attraction, the orbital period depends on orbital radius:

$$T^2=(4\pi^2/GM)r^3$$

Reese Ch. 6A, part 2

(pp. 244-254)

- Geometry of Ellipses
- Kepler's Laws I-III of Planetary Motion
- Newton's Explanation of Kepler's Laws

Kepler's First Law

- The path of a planet orbiting the Sun is an ellipse with the Sun at one focus of the ellipse. (The other focus is empty.)

Geometry of Ellipses

- Definition of an ellipse; the ellipse as one of the conic sections
- Nomenclature
  - Major axis and minor axis
  - Semimajor axis and semiminor axis
  - Foci
  - Planetary ellipses: perihelion and aphelion
  - Eccentricity
- Polar equation:

$$r = \frac{a\left(1 - \varepsilon^2\right)}{1 - \varepsilon\cos\theta}$$

Kepler's Second Law

- The line from the Sun to a given planet sweeps out equal areas during equal time intervals, or …
- The radius vector of a given planet (with the vector's tail fixed at the Sun) sweeps out area at a constant rate.

Newtonian Explanation for Kepler's Second Law

- The Sun's gravitational force is a central force, so the planet's acceleration is always directed toward the Sun.
- The rate at which area is swept out is

$$\frac{dA}{dt} = \left|\frac{1}{2}(\vec{r} \times \vec{v})\right|$$

- But $\vec{a} \parallel (-\vec{r}) \Rightarrow \frac{d}{dt}\left(\frac{dA}{dt}\right) = 0$

Kepler's Third Law

- The square of the orbital period (T) of any planet is proportional to the cube of the semimajor axis (a) of its orbit:

$$T^2 = Ka^3$$

- K is the <u>constant of proportionality</u>: one value works for the entire solar system.

Newtonian Explanation for Kepler's Third Law

- Not difficult to prove under the assumptions that:
  - Planet's mass is much less than Sun's mass
  - Orbit is circular
- A (more challenging) proof can also be obtained when:
  - The two bodies have comparable masses
  - The orbit(s) is(are) elliptical

Newtonian Form of Kepler's Third Law

$$T^2 = \left[\frac{4\pi^2}{G(M+m)} \right] a^3$$

- Newton's form gives us an explicit expression for Kepler's proportionality constant K, allowing us to determine the sum of the masses of the objects.