

Reese Ch. 6, part B (pp. 254-269)

- Astronomical Units
- The Gravitational Field
- Flux of a Vector through a Surface
- Gauss' Law for the Gravitational Field

Astronomical Units

- The semi-major axis of the relative orbit of Earth and Sun is the astronomical unit (AU):
 $1 \text{ AU} = 1.496 \times 10^{11} \text{ m} \approx 500 \text{ light-seconds}$
- The orbital period of Earth's orbit around the Sun is the (sidereal) year:
 $1 \text{ y} = 365.25637 \text{ days} \approx 3.16 \times 10^7 \text{ seconds}$
- Using these units, Kepler's 3rd Law reads:
$$T^2 = (1 \text{ y}^2/\text{AU}^3)a^3$$

The Gravitational Field: I

- At any point in space, the gravitational force that would be experienced by a low-mass test particle placed there is proportional to the test particle's mass. The ratio of force to mass is independent of the test particle's mass: we call that ratio the gravitational field at that location.

The Gravitational Field: II

- According to Newton's 2nd Law, the acceleration of any object is just the resultant force divided by the object's mass. This means that if the only forces acting on a particle are gravitational, then the particle's acceleration will just be the gravitational field at that particle's location!

$$\vec{a}_{\text{testparticle}} = \frac{\vec{F}_{\text{grav}}}{m} = \vec{g}(\vec{r})$$

Flux of a Vector through a Surface: I

- For any vector field, the (infinitesimal) flux of that vector through an infinitesimal surface element is the dot product of the vector and the surface element. For the gravitational field,

$$d\Phi \equiv \vec{g} \cdot d\vec{S} = \vec{g} \cdot \hat{n} dS = g dS \cos \theta$$

Flux of a Vector through a Surface: II

- The flux of the gravitational field through a (finite) surface is the sum (integral) of the infinitesimal contributions from various parts of the surface:

$$\Phi = \int d\Phi \equiv \int \vec{g} \cdot d\vec{S} = \int \vec{g} \cdot \hat{n} dS = \int g dS \cos \theta$$

Gauss' Law for the Gravitational Field: I

- Because the gravitational field is an attractive inverse-square field, it can be shown that the flux of the gravitational field (of an isolated point mass M) through any closed surface that surrounds the point mass is

$$\Phi_{\text{closed surface containing } M} = -4\pi GM$$

- For any closed surface NOT containing M , the flux is zero.

Gauss' Law for the Gravitational Field: II

- Using the law of superposition of fields, we conclude that Gauss' law holds not just for an isolated point mass, but for any distribution of matter whatsoever. The flux of the gravitational field through ANY closed surface satisfies

$$\Phi = -4\pi GM_{\text{enclosed}}$$

- As with electrostatic fields, Gauss' law can be used to easily find the gravitational field of highly symmetric distributions of mass.