Reese Chs. 7 & 8
(pp. 301-302 and 335-352)

- Motion in a Smooth Tunnel through Earth
- Gravitational Potential Energy
- Work-Energy Theorem
- Escape Speeds and Black Holes
- Limitations of Work-Energy Theorem

Motion in a Smooth Tunnel: I

- We imagine drilling a straight and smooth-walled tunnel along a diameter of a (hypothetical) uniform-density Earth.
- Applying Newton's inverse-square gravitation and his 2nd Law of Motion, we find that a rock released from rest at one end of the tunnel executes simple harmonic motion about Earth's center!

$$T = 2\pi \left(\frac{R_{\oplus}^{3}}{GM_{\oplus}} \right)^{\frac{1}{2}}$$

Motion in a Smooth Tunnel: II

- This period is exactly the same as the orbital period of an Earth-skimming satellite – about 84 minutes.
- Now imagine a straight smooth tunnel between two points on Earth's surface that are not at opposite ends of a diameter. Analysis gives SHM w/ same period as before!
- Hence "through the Earth in 42 minutes"!

Motion in Smooth Tunnel: III

- What happens if we are (slightly) more realistic and allow for the fact that the density increases toward Earth's center?
- Tunnel through Earth's center:
  - Is period less than, equal to, or greater than 84 minutes?
  - Is motion still simple harmonic motion?
- What about off-center tunnels?

Work (Review)

- In PHYS 240, we defined work done by a force along a given path as

$$W_{path} \equiv \int_{path} F\cos\theta dl = \int_{path} \vec{F}\cdot d\vec{r}$$

- In general, the work done getting from point P_i to P_f depends on the path.

Conservative Forces (Review)

- For certain forces, the work done by the force as a particle moves from one point to another is independent of the path. These forces are called **conservative forces.** The work done by a conservative force is zero for any closed path: $\oint_{anyclosedpath} \vec{F}_{conservative}\cdot d\vec{r} = 0$

Additional Results I (Review)

- Net Work- Kinetic Energy Theorem: The work done by the net force on a particle equals the change in the particle's kinetic energy.
- When several forces are acting, the net work equals the sum of the individual works.
- For a conservative force, we can define a function of position called the potential energy. We begin by choosing a reference position P_0 where the potential energy is zero.

$$PE_a\left(\vec{r}\right) \equiv -\int_{\vec{r}_0}^{\vec{r}} \vec{F}_a\left(\vec{r}^{\,'}\right)\cdot d\vec{r}^{\,'}$$

Additional Results II (Review)

- Notice the minus sign in the definition of potential energy. This means that the change in potential energy as the particle goes from P_i to P_f is equal to MINUS the work done by the conservative force:

$$PE_a\left(\vec{r}_f\right) - PE_a\left(\vec{r}_i\right) = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_a\left(\vec{r}^{\,'}\right)\cdot d\vec{r}^{\,'}$$

Additional Results III (Review)

- Putting this all together, we find that if a particle is subject to some conservative forces (*a,b,c,..*) and some non-conservative forces (e.g, friction), then the net-work kinetic energy theorem implies that

$$KE_f - KE_i = W_{cons} + W_{non-cons}$$

$$\Rightarrow KE_f + \sum_{a,b,c,..} PE\left(\vec{r}_f\right) = KE_i + \sum_{a,b,c,..} PE\left(\vec{r}_i\right) + W_{non-cons}$$

Gravitational Potential Energy

- For a mass M at the origin, the Newtonian gravitational force on a test mass m at location $\vec{r}$ is

$$\vec{F}\left(\vec{r}\right) = -\frac{GMm}{r^2}\hat{r}$$

- It's not hard to show that this is a conservative force, and if we choose the location of zero potential energy to be r = r_0, then the gravitational potential energy function is

$$PE(\vec{r}) = -GMm\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

Gravitational Potential of a System consisting of Earth and an Object near Earth's Surface: I

- For this system, the potential energy function is:

$$PE(\vec{r}) = -GM_{\oplus}m\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

- If we choose the reference location P_0 to be the Earth's surface and we write *r* in terms of the object's altitude h, we can write the potential energy of the system as:

$$PE(h) = -GM_{\oplus}m\left[\left(\frac{1}{R_{\oplus}+h}\right) - \left(\frac{1}{R_{\oplus}}\right)\right]$$

Gravitational Potential of a System consisting of Earth and an Object near Earth's Surface: II

- Escape?: Notice for later use that the change in potential energy going from h = 0 to h→∞ is

$$\Delta(PE) = +\frac{GM_{\oplus}m}{R_{\oplus}}$$

- Low-altitude approximation: However, if h<< (radius of Earth), then

$$PE(h) = -\left(\frac{GM_{\oplus}m}{R_{\oplus}}\right)\left[\left(1+\frac{h}{R_{\oplus}}\right)^{-1} - 1\right] \approx \frac{GM_{\oplus}m}{R_{\oplus}^{\ 2}}h = mg_{\oplus 0}h$$

Escape from Earth: I

- Imagine launching a projectile vertically from an idealized Earth (airless, nonrotating, spherical, isolated). What minimum initial speed gives escape?
- Apply the conservation of mechanical energy, taking final radius as infinite and final KE as 0:

$$\frac{1}{2}mv_f^2 + PE_\infty = \frac{1}{2}mv_i^2 + PE_i \Rightarrow 0 + (PE_\infty - PE_i) = \frac{1}{2}mv_{escape}^2$$

$$\Rightarrow v_{escape} = \sqrt{\frac{2GM_\oplus}{R_\oplus}}$$

Escape from Earth: II

- How does the minimum speed for escape depend on the launch direction?
  - It doesn't! Although the trajectory would be different, the final outcome is still escape.
- Suppose that an object is already in an Earth-skimming orbit. How much must its speed be increased to cause escape?

$$\Delta v_{required} = v_{escape} - v_{loworbit} = (\sqrt{2} - 1)v_{loworbit}$$

  - The satellite's speed needs a 41.4% boost.

Escape from Anywhere? Nope!

- According to Newton, we can escape from the surface of Planet X as long our launch speed exceeds

$$v_{escX} = \sqrt{\frac{2GM_X}{R_X}}$$

- But Newton's gravity is inconsistent with Einstein's 1905 special relativity. Einstein's 1915 replacement gravity theory (general relativity) implies that nothing (not even light!) can escape from an object of mass M that has a radius less than

$$R_S \equiv \frac{2GM}{c^2}$$

Limitations on Use of Work-Energy Theorem

- For systems consisting of one or several simply pointlike particles for which there are no internal motions, deformations, or thermal effects, the work-energy theorem is a very useful tool.
- In systems with internal motions, deformations, and thermal effects, the work-energy theorem doesn't hold, although Newton's 2nd Law still applies to each part.
- Physicists do believe in and use <u>conservation of **total** energy</u>, but there are energy exchanges not covered by our macroscopic mechanics of rigid particles. See Reese pp. 351-352; for energy conservation, see Reese Ch.13.