

Reese Ch. 11 (pp. 489-516)

- States of Matter
- Stresses, Strains, and Elastic Moduli
- (Some Review Slides on Fluids)
- Continuity Equation for Fluid Motions
- Bernoulli's Principle
- Viscosity and Viscous Flows

States of Matter

It is common to describe samples of bulk matter as belonging to one of several states (solid, liquid, gas, or plasma), even though for many materials and conditions of temperature and pressure, matter can exhibit combinations of characteristics that don't fit neatly into this scheme.

Stress and Strain

- Generically, ***stress*** refers to a deforming force (per unit area), while ***strain*** refers to the corresponding deformation. Strain is typically quantified in a form that is dimensionless.
- Then an elastic material can be characterized by various elastic constants using the following generic equation:
$$\text{stress} = (\text{elastic "modulus"}) \times (\text{strain})$$

Elasticity in Length: I

If an elastic object of relaxed length L_0 and cross section A is subjected to a stretching force $\mathbf{F}$ (actually a pair of forces of magnitude F) applied at the ends of the object, the ***tensile stress*** is F/A . If the object is stretched by an amount ΔL , its ***tensile strain*** is $(\Delta L)/L_0$, which is dimensionless.

Elasticity in Length: II

Using the generic definition of the elastic constant

elastic modulus = stress / strain

we define the **Young's modulus Y** for the material as

$$Y = (F/A)/(\Delta L/L)$$

Note: When $F > 0$ and $\Delta L > 0$, the object is under **tension**; when F and ΔL are negative, the object is under **compression**.

Elasticity in Volume

If an object made of an isotropic elastic material is subjected to compression on all sides, then the object's volume decreases. The compressive stress $|\mathbf{F}|/A$ is denoted by ΔP , and the volume strain is $(\Delta V)/V$. To get a positive elastic modulus, we write

$$B = - \Delta P/(\Delta V/V)$$

B is called the **bulk modulus**

Elasticity of Shape: I

If opposing forces are applied on opposite sides (of area A) of an elastic rectangular object with the forces in the plane of those sides, we say the object is subject to a **shear stress** F/A . If the sides are a distance h apart and the *tangential relative displacement* of points on the opposite sides is Δx , the **shear strain** is defined as $\Delta x/h$.

Elasticity of Shape: II

Then our generic rule gives:

$$S = (F/A)/(\Delta x/h)$$

S is called the **shear modulus**

Stretch of a Spring: Shear Strain

When a force is applied to stretch (or compress) a coil spring, the stretching is actually the result of shear strain in the spring material: the “pitch” of the helix is increased, as the wire that makes up the string is twisted. Careful application of the theory of elasticity provides a formula for **k**, the spring constant.

Spring Constant for Hooke's Law

For a helical coil spring that is “closely wound” (i.e., has a pitch angle is much less than 1 radian), the spring constant **k** is given by:

$$k = \frac{S r^4}{4 N R^3}$$

Here **S** is the shear modulus of the material, **r** is the radius of the wire, **R** is the radius of the coil, and **N** is the total number of turns in the coil.

*Notice that you can find **S** from **k**, **r**, **R**, and **N**.*

Solids, Liquids, and Gases

- The simplest way to describe the differences among these three “states of matter” is to say:
 - A solid has a definite volume and shape.
 - A liquid has a definite volume but no definite shape.
 - A gas has neither a definite volume nor a definite shape.
- Liquids and gases are collectively referred to as **fluids**. In an elastic solid, the strain is proportional to the stress. In a fluid, the **rate of strain is proportional to the stress.**

Fluid as a Continuum: What about Atoms?

We describe a fluid by using functions that depend on location and time -- quantities such as density, pressure, temperature, and fluid velocity. But all matter is made up of atoms! Why (and when) can we use the “fluid approximation?” When does it fail?

Mass Density

The mass density function $\rho(\mathbf{r}, t)$ of a fluid is defined as the small-volume limit of the instantaneous mass to volume ratio of a small region of space surrounding the location $\mathbf{r}$. We need to imagine that ΔV is small enough that cutting it in half won't change the ratio, but also large enough to contain many atoms, so that cutting it in half won't change the ratio!!

Densities of Liquids

Liquids are difficult to compress (remember, they have a "definite" volume), so that over a wide range in pressure we can talk about THE density of the liquid. (This has its limits: water can boil and freeze ***simultaneously*** at low pressures!)

Caution: Most liquids DO exhibit a noticeable variation of density with temperature.

Compress Water? Not much!

Recall the equation of volume elasticity:

$$B = - \Delta P / (\Delta V / V)$$

- Here ΔP is the added pressure, and the volume strain is $(\Delta V)/V$, and B is the ***bulk modulus***.
- For water, B is roughly $0.2 \times 10^{10} \text{ N/m}^2 = 2 \times 10^4 \text{ atm}$. Even at the bottom of the Marianas Trench, where the added pressure is about 10^3 atm , the water is only compressed by about 5%!

Densities of Gases

The volume occupied by a fixed mass of gas completely depends on the ***confining pressure*** supplied by the environment. (Remember, gases have no definite volume.)

At a fixed temperature, the pressure required to confine a dilute gas is proportional to its density. (Boyle's Law)

Details: The Ideal Gas Law

When a gas is sufficiently warm and dilute, the ideal gas law relates pressure, density, temperature, and composition:

$$PV = nRT \quad \text{and} \quad n = m/M$$

$$\mathbf{P = [(m/V)RT]/M = \rho RT/M}$$

With a gas, if you change the pressure by a factor of 1000 and hold T constant, you change the volume by a factor of 1000!

Fluid Pressure

- A fluid can **only** be in static equilibrium in the presence of a stress that is isotropically compressive (or, in certain situations, tensile). This is because any “shear stress” would produce continuing fluid deformation. The compressive stress is called ***static pressure***.

Fluid Equilibrium and Gravity

A horizontal “slab” of fluid of density ρ and thickness Δh has a mass per unit area $\rho\Delta h$ and weight per unit area $\rho g\Delta h$. Thus there can only be static equilibrium if the fluid pressure increases with depth h (which increases downward): $\Delta P = \rho g\Delta h$, or . . .

$$dP/dh = \rho g$$

This is called the ***barometric equation***.

Pascal’s Law

If we increase the surface pressure of a liquid (for example, by compressing the gas that’s above the liquid surface), the barometric equation guarantees that **the extra surface pressure produces the same amount of added pressure throughout the fluid**.

This result is known as ***Pascal’s Law***, after the 17th-century scientist/philosopher.

Pressure Measurements

- Pressure-measuring instruments include the ***open-tube manometer*** and ***the mercury barometer***.
- It is important to understand the distinction between absolute pressure and gauge pressure. Gauge pressure is the ***difference*** between the pressure being measured and the “outside world” (Earth’s atmosphere).

Buoyancy of Submerged Objects

The pressure increases with depth in a fluid in static equilibrium, so there is an ***upward pressure force acting on a submerged (or partially submerged) object equal to the weight of displaced fluid***. This is known as **Archimedes’ Principle** after the Greek mathematician/scientist/engineer of the 3rd Century BC. This principle can be used to analyze and explain many situations involving immersion of an object in a fluid.

Fluid Flow Characteristics: I

- **Steady versus unsteady flows:** A fluid flow is steady if there is a frame of reference in which all of the flow variables are independent of time.
- **Rotational versus irrotational flows:** A fluid flow is locally irrotational if there is no location within the fluid where a small spin meter would be set spinning by the fluid.

Fluid Flow Characteristics: II

- **Compressible vs. Incompressible Flows:** A fluid flow is incompressible if every element of the fluid moves without changing its density.
- **Viscous vs. Inviscid (Nonviscous) Flows:** A fluid flow is inviscid if the effects of internal fluid friction can be neglected.

Fluid Flow Characteristics: III

- **Laminar versus Turbulent Flows:** In laminar flow, fluid particles move along relatively smooth curves that are essentially parallel to one another and do not mix. It is typical of relatively low-speed, small-scale flow of viscous liquids. In turbulent flow, there is significant disordered motion of fluid particles in directions transverse to the main flow direction.

Fluids: Additional Terminology

- Path Line: This is the trajectory of a particular fluid element or particle.
- (Instantaneous) Streamline: This is the curve which is tangent to the instantaneous velocity vectors at all points along its length.
- Streak Line: This is the instantaneous locus of all fluid elements that at (various) earlier moments passed through a specified location.
- **ACHTUNG!** Only in steady flow do these three sets of curves coincide.

Lagrangian vs. Eulerian Descriptions

- In the Lagrangian description, we follow the motion of a specific fluid element. It is easiest to think about how to express Newton's Second Law in this description. (*Each fluid particle carries a "Walkman transmitter," and we do our accounting using the serial numbers on each Walkman.*)
- In the Eulerian description (which is much more commonly used), we regard all of the variables as functions of $\mathbf{r}$ and t . For example, $\mathbf{v}(\mathbf{r}, t)$ is the instantaneous velocity at location $\mathbf{r}$ of whatever fluid element happens to be passing through that place at that moment. (*Fixed radio transmitters.*)

Equation of Continuity

The equation of continuity expresses the conservation of matter. It is one of the equations in the toolbox used in analyzing fluid motion. It is a precise statement of the fact that an expanding blob of fluid exhibits a density decrease, while a contracting blob of fluid exhibits a density increase.

Steady Flow of an Ideal Fluid: Bernoulli's Equation

- NOTE: An ideal fluid is one in which we can neglect both viscous forces and heat conduction.
- In 1738, Daniel Bernoulli obtained an very useful equation that is applicable to the steady flow of an ideal fluid. It is based on Newton's Second Law, and it states that **the following quantity is constant along a streamline:**

$$P + (1/2)\rho v^2 + \rho U + \rho \Phi$$

- If the fluid is of uniform density and the flow is also incompressible and irrotational, then the above quantity is constant throughout the **entire fluid**. In this case, U will also be separately a constant, so we have . . .

Bernoulli's Equation

$$P + (1/2)\rho v^2 + \rho gh = \text{constant}$$

This equation can help us understand how airplanes fly and why curveballs curve.

Viscosity and Viscous Flow: I

- A wide variety of experiments have shown that at the boundary between a fluid and a stationary solid surface, the fluid must also be at rest. This is called the "no-slip condition."
- Wherever two neighboring streams of fluid flow past one another, each one exerts a force on the other, tending to reduce the relative motion.

Viscosity and Viscous Flow: II

- The property of the fluid that quantifies this internal stickiness/friction ("sticktion") is called viscosity. The coefficient of viscosity is defined by the equation

$$\eta \equiv \frac{(F/A)}{(dv_x/dy)}$$

- Viscous drag is the cause of the downstream pressure drop in relatively low-speed flows through small-diameter channels.