Reese Ch. 12 (portions)
(pp. 558-559, 565-574)

- Traveling Waves (Review)
- Principle of Superposition (Review)
- Standing Waves (Review)
- Wave Groups and Beats
- Fourier Synthesis and Analysis
- The Uncertainty Principle(s)

Traveling Waves: I

- A generic wave traveling in the +x direction is mathematically described by f(x-vt), where v is the wave speed, and the function f gives the form of the wave.
- A generic wave traveling in the –x direction is mathematically described by g(x+vt).

Traveling Waves: II

- A periodic traveling wave is one for which the wave form (at all x) at time $t_2 = t_1 + T$ is exactly the same as it was at time t_1. T is called the period of the wave.
- A special kind of periodic traveling wave is a sinusoidal traveling wave:

$$\Psi(x,t) = A\cos[\frac{2\pi}{\lambda}(x - vt) + \phi_o]$$

- Questions:
  - How is T related to v and λ?
  - What do the quantities A and ϕ_o represent?
  - In which direction is the wave given above traveling?

Describing Sinusoidal Waves

- Amplitude A
- Phase constant ϕ_o
- Wavelength λ
- Wave number k
- Period T
- (Ordinary) frequency f *(or ν)*
- Angular frequency ω

Principle of Superposition

If the basic principles (such as Newton's laws for mechanical waves or Maxwell's equations for electromagnetic waves) lead to linear equations for a wave disturbance, then whenever two or more wave disturbances arrive at the same location, the resultant disturbance is simply the sum of the individual disturbances:

$$\Psi_{total}(x,t)=\Psi_1(x,t)+\Psi_2(x,t)+\ldots+\Psi_N(x,t)=\sum_{i=1}^{N}\Psi_i(x,t)$$

Sinusoidal Standing Waves: I

When two sinusoidal traveling waves of the same amplitude and frequency (but opposite direction of propagation) are superposed, a standing wave results:

$$\Psi_1(x,t) = A\cos(kx-\omega t)$$
$$\Psi_2(x,t) = A\cos(kx+\omega t)$$
$$\Rightarrow \Psi_{total}(x,t) = 2A\cos(kx)\cos(\omega t)$$

Sinusoidal Standing Waves: II

- Various structures (strings, open and closed pipes, bars, etc.) have specific natural frequencies at which they can support standing waves. (Not all structures sustain standing waves that are sinusoidal in their *spatial* dependence, but all standing waves ARE sinusoidal in their time dependence.)
- The study of these structures and the sounds they produce is acoustics. Acoustics provides explanations of many musical phenomena.

Wave Groups and Beats: I

If two sinusoidal waves of equal amplitudes but slightly different frequencies are traveling in the same direction, the principle of superposition yields:

$$\Psi_{total}(x,t)=2A\cos(kx-\omega t)\cos\left[\left(\frac{\Delta k}{2}\right)x-\left(\frac{\Delta \omega}{2}\right)t\right]$$

Wave Groups and Beats: II

- This means that at a fixed location, there is (roughly speaking) a "fast" oscillation of (angular) frequency ω whose amplitude slowly rises and falls with angular frequency $\Delta\omega$. (Why not $\Delta\omega/2$?)
- At fixed time, the spatial variation is (roughly speaking) a sinusoid with wave number k whose amplitude gradually varies with location (maxima are separated by $\Delta x = 2\pi/\Delta k$).

Wave Groups and Beats: III

Phase velocity: The peaks of the short-wavelength oscillations move with velocity

$$v_{phase} = \frac{\omega}{k}$$

Group velocity: The long-wavelength pattern (the envelope) moves with velocity

$$v_{group} = \frac{\Delta\omega}{\Delta k}$$

Wave Groups and Beats: IV

- If the phase velocity is the same for both frequencies (as in the case of sound waves), then the waveform travels without changing its shape.
- If $\Delta\omega$ is small enough, a listener hears a sound of the (average) frequency which varies in loudness. The number of loudness maxima heard per second is called the number of beats per second and is equal to the difference between the (ordinary) frequencies of the two waves:

$$\left|\frac{\Delta\omega}{2\pi}\right| \equiv |\Delta\nu| \equiv |\nu_2 - \nu_1|$$

Fourier Synthesis and Analysis

- The French mathematician Fourier realized that any well-behaved periodic function can be expressed as a sum (in general, an infinite series) of sinusoids whose frequencies are integer multiples of the fundamental frequency (1/T). Building up a function this way is called Fourier synthesis.
- Fourier developed a method for working from the function itself to find what values to use for the coefficients. This is called Fourier analysis.
- Fourier series can only represent periodic functions. However, an extension of this idea, using the whole continuous range of frequencies, is used to represent functions that are not periodic, such as wave pulses. This technique is called Fourier transformation.

The Uncertainty Principles: I

- In the superposition of two different sinusoids, the spatial "width" Δx of each long maximum is inversely related to Δk, the difference between the wave numbers of the two waves. This is simply a consequence of the mathematics of adding two slightly different sinusoidal functions.
- This inverse relationship also applies to the infinite sums (integrals) used when Fourier transforms are applied.
- As applied to the spread in wave numbers and the width of the corresponding wave function Ψ, the uncertainty principle states that

$$\boxed{\Delta k \Delta x \geq \frac{1}{2}}$$

The Uncertainty Principles: II

- When a function of time is expressed as a superposition of sinusoids of various frequencies, a completely analogous relationship holds. The frequency-time uncertainty principle states that

$$\boxed{\Delta \omega \Delta t \geq \frac{1}{2}}$$

- To put it in musical terms, a note of duration Δt must involve a range of angular frequencies $\Delta\omega$ that's at least $\frac{1}{2\Delta t}$.

The Uncertainty Principles: III

- These inverse relationships were well known before 1850 to people who specialized in wave physics or applied mathematics.
- The name uncertainty principles is more recent (late 1920's), and is associated with the German physicist Werner Heisenberg, one of the founders of quantum mechanics.
- According to quantum mechanics, matter exhibits wave-particle duality. Specifically, the momentum of a particle is proportional to the wave number of an associated wave. Similarly, the energy of a particle is proportional to the frequency of an associated wave.

The Uncertainty Principles: IV

- Mathematically, Heisenberg's uncertainty principles are:

$$\boxed{\Delta p \Delta x \geq \frac{\hbar}{2}}$$ (position-momentum u.p.)

$$\boxed{\Delta E \Delta t \geq \frac{\hbar}{2}}$$ (energy-time u.p.)

where $\hbar = 1.055 \times 10^{-34}\, Joule \cdot \sec$

(NOTE: This is described in more detail in Reese, Chapter 27.)