Reese Ch. 14, part A
(pp. 639-649)

- Kinetic Theory of Gases
- The Ideal Gas Approximation
- Interpretation of Gas Pressure
- The Meaning of the Absolute Temperature
- Internal Energy and Specific Heats

Assumptions of Classical Kinetic Theory of an Ideal Gas: 1-2-3

1. The number of molecules N in the gas is very large.
2. The volume V of the gas is much larger than the total volume occupied by the particles themselves: $V >> NV_{molec}$.
3. The molecules obey Newton's laws.

Assumptions of Classical Kinetic Theory of an Ideal Gas: 4-5-6

4. A molecule is equally likely to be moving in any direction.
5. The molecules interact with each other and with the walls of the container only via elastic collisions.
6. The gas is in thermal equilibrium with its surroundings.

Assumptions of Classical Kinetic Theory of an Ideal Gas: 7

7. The molecules of the gas are identical and indistinguishable.

NOTE:

This last assumption means that the initial treatment is applicable only to an isotopically pure gas of a single element or compound. However, the treatment is easily modified to describe mixtures.

Pressure of an Ideal Gas

- By carefully considering the elastic collisions of individual molecules with an area A of the container wall, we find that the time-averaged rate of momentum transfer is directed along the outward normal: $\vec{F} = \hat{n}\left(\frac{N\mu}{3V}\right)\langle v^2\rangle A$
- Therefore the pressure (outward normal force per unit area) is given by:

$$\boxed{P = \left(\frac{N\mu}{3V}\right)\langle v^2\rangle = \left(\frac{2N}{3V}\right)\left[\frac{1}{2}\mu\langle v^2\rangle\right]}$$

Meaning of the Absolute Temperature

- Kinetic theory gives: $\boxed{PV = \frac{2N}{3}\left(\frac{1}{2}\mu\langle v^2\rangle\right)}$
- The ideal gas law is: $\boxed{PV = NkT}$
- This implies that: $\boxed{\frac{1}{2}\mu\langle v^2\rangle = \frac{3}{2}kT}$

Molecular speeds

- The root-mean-square (rms) speed is easily found:

$$\boxed{v_{rms} \equiv \sqrt{\langle v^2\rangle} = \sqrt{\frac{3kT}{\mu}} = \sqrt{\frac{3RT}{M}}}$$

- The average speed is less than the rms speed. Using the Maxwellian distribution: $\boxed{\langle v\rangle = \sqrt{\frac{8kT}{\pi\mu}} \approx 0.92v_{rms}}$

  and the most probable speed is: $\boxed{v_{mp} = \sqrt{\frac{2kT}{\mu}} \approx 0.82v_{rms}}$

Ideal Gas Mixtures

- The preceding implies that two different pure gases at the same temperature have the same average translational kinetic energy per molecule. We expect (and experiments confirm) in a gas mixture at a certain temperature T, the average molecular translational kinetic energies are equal.
- Furthermore, the pressure exerted by an ideal gas mixture is just the sum of the pressures that each gas by itself would exert.

$$\boxed{P = \sum_i P_i = \sum_i \frac{N_i kT}{V} = \frac{\left(\sum_i N_i\right)kT}{V}}$$

This is called the law of partial pressures.

Internal Energy and Specific Heat of a Monatomic Ideal Gas

- The internal energy of a monatomic ideal gas is just the sum total of the translational kinetic energies of the N molecules: $\boxed{U = N\left(\frac{3}{2}kT\right) = \frac{3}{2}NkT}$
- If a gas is heated at constant volume, it does zero work, so C_V, the molar specific heat at constant volume of a monatomic ideal gas, is: $\boxed{c_V = \frac{3}{2}N_A k = \frac{3}{2}R}$

Heating an Ideal Gas at Constant Pressure

- If an ideal gas is heated at constant pressure, it expands and does work. This means that the amount of heat transfer needed to produce a given temperature rise is greater for isobaric heating than for isochoric heating: $\boxed{c_P = c_V + R}$
- The above equation holds for any gas (monatomic or not) whose internal energy only depends on its temperature.

Specific Heats for Diatomic and Polyatomic Gases

- The molar specific heat at constant volume for a diatomic or polyatomic gas is greater than 3R/2.
- This indicates that when such a gas is heated, energy is being transferred not only into translational kinetic energy of the molecules, but also into forms of energy internal to the molecules.