

Reese Ch. 16, Part A (pp. 733-744)

- Review: Coulomb's Law
- Review: Electric Field & Superposition
- Electric Field due to Continuously Distributed Charges
- Motion in Uniform Electric Fields
- Gauss's Law for Electric Fields

Review: Coulomb's Law: I

- Between 1600 and 1751, investigators such as Gilbert, Dufay, and Franklin established that there are two kinds of charge and that like charges repel one another while unlike charges attract.
- In the 1780's, about a century after Newton published the *Principia* but more than a decade before Cavendish's experiment to determine G, Charles Coulomb, established that concentrated electric charges interact via an inverse square force.

Review: Coulomb's Law: II

- Using modern units, this can be summarized by writing

$$|F_{elec}| = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{|\vec{r}_2 - \vec{r}_1|^2}$$

- The constant ϵ_0 is called the "permittivity of free space." In SI units, the coefficient in Coulomb's law is $\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2}$

Review: Electric Field

- In the neighborhood of a fixed charge Q, the force on a concentrated ("point") charge is proportional to its electric charge q. For this and other reasons, it is very useful to think of Q as producing a "field" around it.
- At any given location $\mathbf{r}$, this field is defined as the ratio of the force to charge that Q exerts on a positive "test" charge q:

$$\vec{E}_{duetoQ}(\vec{r}) \equiv \frac{1}{q} \left(\frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \right) \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

- NOTES:
 - (1) We have located the origin at Q for simplicity.
 - (2) The field points radially outward (inward) if $Q > 0$ ($Q < 0$).

Review: Superposition

- Experiments show that electric forces obey the law of superposition: the force on a test charge in the presence of several fixed charges is given by the vector sum of the "individual" forces.
- Because the electric field is just the force divided by the test particle's charge, this superposition also applies to the electric field. Given an array of fixed charges q_n at locations $\vec{r}_n$, the electric field is:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_o} \sum_n \frac{q_n(\vec{r} - \vec{r}_n)}{|\vec{r} - \vec{r}_n|^3}$$

Electric Field Due to Continuously Distributed Charges

- In many practical situations, electric fields are produced by very many charges that are individually of small magnitude. (Avogadro's number is 6.02×10^{23} & the elementary charge is 1.60×10^{-19} coulombs!)
- In this case, the superposition law becomes an integral over all of the contributing charge:

$$\vec{E}(\vec{r}_p) = \frac{1}{4\pi\epsilon_o} \int \left[\frac{dq}{|\vec{r}_p - \vec{r}_{dq}|^2} \frac{\vec{r}_p - \vec{r}_{dq}}{|\vec{r}_p - \vec{r}_{dq}|} \right]$$

- The only way to really understand this is to practice!!

Motion in Uniform Electric Fields

- If no magnetic fields are present and if we ignore the tendency of the test charge to radiate as it accelerates, a test particle moving in an electrostatic field (the field created by given fixed charges) is subject to an instantaneous force $q\vec{E}[\vec{r}(t)]$, so Newton's 2nd Law gives:

$$\vec{a} = \frac{q}{m} \vec{E}[\vec{r}(t)]$$

- Note that if the electric field is uniform, then the test particle exhibits **constant acceleration**, so that

$$\vec{r}(t) = \vec{r}_o + \vec{v}_o t + \frac{q\vec{E}_o}{2m} t^2$$

Gauss's Law for Electric Fields

- By analogy with the gravitational case, we should expect that the net flux of the electric field through any closed surface is proportional to the net electric charge in the volume enclosed by the surface:

$$\int_{\substack{\text{closed} \\ \text{surface} \\ S}} \vec{E} \cdot d\vec{S} = \frac{Q_{\text{net enclosed}}}{\epsilon_o}$$

- For highly symmetric charge distributions, Gauss' law can be used to determine the electric field pattern.

Reese Ch. 16, Part B (pp. 744-751)

- Using Gauss's Law to Find Electric Fields
 - Single Isolated Point Charge
 - Infinitely Long Uniformly Charged Line
 - Inside a Uniformly Charged Solid Sphere
 - Uniformly Charged Infinite Sheet
- Conductors: Electric Fields and Charge
- Semiconductors and Superconductors

Gauss's Law: Single Isolated Point Charge

- Spherical symmetry
 - Rationale for radially directed field
 - Rationale that field strength depends only on r
- What Gaussian surface to choose?
 - A centered spherical surface (obviously!?)
 - Then

$$\int_{\text{closed surface}} \vec{E} \cdot d\vec{S} = (\vec{E} \cdot \hat{r}) \times (4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{r^2} \hat{r}$$

Gauss's Law: Infinitely Long Uniformly Charged Line: I

- Cylindrical Symmetry
 - Rationale for zero azimuthal field
- Infinite Length and Uniform Charge
 - Rationale for field strictly "radial"
 - Rationale for field strength only depending on the "perpendicular radius" r

Gauss's Law: Infinitely Long Uniformly Charged Line: II

- Gaussian surface: coaxial circular cylinder
- Then

$$\int_{\text{closed surface}} \vec{E} \cdot d\vec{S} = (\vec{E} \cdot \hat{r}) \times (2\pi r \ell) + 2 \times 0 \times (\pi r^2) = \frac{\lambda \ell}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{2\lambda}{r} \hat{r}$$

Gauss's Law: Uniformly Charged Solid Sphere

- Spherical symmetry
 - Rationale for radially directed field
 - Rationale that field strength depends only on r
- Gaussian surface: concentric sphere
 - If $r > R$, then $\vec{E} = \left(\frac{1}{4\pi\epsilon_o} \right) \frac{Q}{r^2} \hat{r}$ but if $r < R$

$$\int_{\text{closed surface}} \vec{E} \cdot d\vec{S} = (\vec{E} \cdot \hat{r}) \times (4\pi r^2) = \frac{Q_{\text{encl}}}{\epsilon_o} \Rightarrow \vec{E} = \left(\frac{1}{4\pi\epsilon_o} \right) \frac{Qr}{R^3} \hat{r}$$

Gauss's Law: Uniformly Charged Infinite Sheet: I

- Planar symmetry (with sheet = the xy plane)
 - Rationale for field along $\pm z$ axis
 - Rationale that strength could only depend on $|z|$
 - Rationale that field either points "outward" on both sides or "inward" on both sides

Gauss's Law: Uniformly Charged Infinite Sheet: II

- Choose Gaussian surface to be a right cylinder with axis along z and bisected by xy plane. Then

$$\int_{\text{closed surface}} \vec{E} \cdot d\vec{S} = 0 + \left[(\vec{E}_{\text{above}} \cdot \hat{k}) + (-\vec{E}_{\text{above}} \cdot -\hat{k}) \right] (A_{\text{end}}) = \frac{\sigma A_{\text{end}}}{\epsilon_o}$$

$$\Rightarrow \vec{E}_{\text{above}} = \frac{\sigma}{2\epsilon_o} \hat{k}$$

- N.B.: Field strength INDEPENDENT of $|z|$!!

Conductors

- A conductor is a material in which there are plenty of charge carriers (electrons) that can quickly move in response to any *macroscopic* electric field.
- This rapid internal rearrangement of charge "shorts out" any applied electric field, so ...
- The (macroscopic) electrostatic field MUST BE ZERO everywhere inside a conductor.

The Interior of a Conductor

- Consider any (macroscopic) region of space whose surface lies inside a conductor.
- We apply Gauss's Law to this region and conclude that the net charge enclosed is zero.
- Since this is true for any region in the conductor's interior, we conclude that the electric charge density must be zero everywhere inside a conductor.
- In a conductor that is charged and or polarized, the only place where charges can reside is on the surface of the conductor.

Electric Field Just Outside the Surface of a Conductor: I

- The electrostatic field just inside the surface is ZERO.
- The electrostatic field is a conservative field: the line integral of "E dot ds" is zero around any closed path.
 - This implies that the tangential component of the field just inside and the tangential component of the field just outside the surface must have the same value.
- Therefore, the electric field just outside a conductor must be perpendicular to the surface!

Electric Field Just Outside the Surface of a Conductor: II

- The electrostatic field just inside the surface is ZERO.
- Construct a tiny Gaussian pillbox
 - infinitesimal thickness
 - diameter small compared to distance along the surface over which the charge density varies
 - midplane IS the surface
- Then Gauss's Law implies that the field just outside is:

$$E_{normal} = \frac{\sigma_{local}}{\epsilon_0}$$

Semiconductors and Superconductors

- Semiconductors (important examples are germanium and silicon) are materials in which there is a relatively small energy gap (less than 2 eV) between the "valence band" and the "conduction band." At room temperature, these materials are neither good conductors nor good insulators.
- Superconductors are materials which lose all electrical resistance when cooled below a critical temperature T_C . Examples include the element mercury ($T_C = 4.2$ K) and the compound $YBa_2Cu_3O_7$, for which $T_C = 92$ K (which is "warmer" than liquid nitrogen!).