Reese Ch. 17, Part A
(pp. 774-786)

- Review: Electrostatic Force
  and Electrostatic Potential Energy
- Review: Electrostatic Field
  and Electrostatic Potential
- Electric Potential due to
  Continuously Distributed Charges
- Equipotential Surfaces and Volumes
- Field as the Negative Gradient of the Potential
- Motion of a Charge in E Field and the electron-volt

Review: Electrostatic Force and Electrostatic Potential Energy

- The force on a test charge due to a static distribution of other electric charges is a conservative force, so that it is possible to define a corresponding electrostatic potential energy:

$$\boxed{PE(\vec{r}_f) - PE(\vec{r}_i) \equiv -\int_i^f \vec{F}_{elec} \bullet d\vec{r}}$$

- NOTES:
  (1) Note the minus sign in the definition.
  (2) The equation above only determines changes in potential energy. The reference location (at which PE = 0) is a matter of choice.

Review: Electrostatic Field and Electrostatic Potential

- The force on a test particle equals test particle's charge TIMES the local electrostatic field: $\boxed{\vec{F}_{elec} = q\vec{E}}$
- Hence we can define an electrostatic potential V by saying that the change in electrostatic PE is just the test particle's charge times the change in the electrostatic potential. This implies that:

$$\boxed{V(\vec{r}_f) - V(\vec{r}_i) \equiv -\int_i^f \vec{E} \bullet d\vec{r}}$$

- Note minus sign and free choice of reference.

SI Unit for Electric Potential

- The SI unit for electric potential is called the **volt**. It is equal to the SI unit of energy divided by the SI unit of charge:

$$\boxed{1\ volt = 1\ joule/coulomb \quad \Rightarrow \quad 1\ \mathrm{V} \equiv 1\ \mathrm{J/C}}$$

- NOTE: Be aware that the abbreviation for this unit (a capital V) is also commonly used as the variable name for electric potential!

Superposition Law for Electric Potential

- The law of (vector) superposition for electric fields implies a (scalar) superposition law for electric potential.
- NOTE: In order to easily determine the reference (V=0) position when potentials are added, it's good practice to use the same reference position for all terms in the sum. For example, if we have a set of several fixed charges and adopt the choice that V=0 for r→∞, then

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\sum_n \frac{Q_n}{|\vec{r}-\vec{r}_n|}}$$

Electric Potential due to Continuously Distributed Charges

- When the charges are distributed continuously, the superposition law takes the form of an integral:

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{dQ}{|\vec{r}-\vec{r}_{dQ}|}}$$

- RESTRICTION: The above equation can be used only when the charge distribution is confined to a bounded region. With infinitely extended distributions, we must calculate the electric field and then integrate the field to find potential differences.

Equipotential Surfaces and Volumes

- From the definition of the electric potential in terms of the electric field, we find:

$$\boxed{V(\vec{r}+d\vec{r}) - V(\vec{r}) \equiv dV \equiv -\vec{E}(\vec{r})\bullet d\vec{r}}$$

- This means that starting from any position vector **r**, the neighboring points **r + dr** that have the same potential are those for which $\vec{E}\bullet d\vec{r} = 0$:
  - (1) If the local field is nonzero, this defines an equipotential surface.
  - (2) If the local field is zero, then there can be a 3-D equipotential volume.

Field as Negative Gradient of Potential: I

- For any function of position, the directional derivative is defined by

$$\boxed{\frac{df(\vec{r},\hat{n})}{ds} \equiv \lim_{\Delta s\to 0}\left[\frac{f(\vec{r}+\hat{n}\Delta s) - f(\vec{r})}{\Delta s}\right]}$$

- It is tedious but not difficult to show that:

$$\boxed{\frac{df(\vec{r},\hat{n})}{ds} = \hat{n}\bullet\left[\hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}\right] \equiv \hat{n}\bullet\vec{\nabla}f}$$

- $\vec{\nabla}f$ is called the gradient of the function f.

Field as Negative Gradient of Potential: II

- Examining how the electric field is related to the potential function, and
- comparing that with how the gradient of f is related to f, we conclude that the electric field is nothing but the negative gradient of the electric potential:

$$\boxed{\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})}$$

Acceleration of Charges by Electrostatic Fields

- Newton's 2nd Law provides:

$$\boxed{m\vec{a} = \vec{F}_{net} = q\vec{E} = -q\vec{\nabla} V}$$

- It is easy to use this to derive:

$$\boxed{\frac{1}{2}mv_f^2 + qV_f = \frac{1}{2}mv_i^2 + qV_i}$$

An auxiliary energy unit: the electron-volt

- An electron-volt is the kinetic energy increase exhibited by an electron when its electric potential increases by one volt. (Notice that since the electron has a negative charge, it is accelerated in the direction of increasing electric potential – opposite the direction in which the field points.)
- Since the magnitude of the charge on an electron is $1.602 \times 10^{-19} C$, we have

$$\boxed{1\ electron - volt \equiv 1\ \mathrm{eV} = 1.602 \times 10^{-19} J}$$

Reese Ch. 17, part B (pp. 786-795)

- Review: An Electric Dipole & its Field (pp. 728-733)
- An Electric Dipole in an External Field
- Electric Potential due to a Dipole
- Using Potential to find the Field of a Dipole
- Potential Energy of an Assembly of Point Charges
- Lightning Rods

Review: An Electric Dipole and Its Field

- An electric dipole consists of two charges of equal magnitude |Q| but opposite sign separated by a relatively short distance d.
- The dipole moment **p** of a dipole is a vector with magnitude |Q| times d, located midway between the charges, and pointing from the negative charge toward the positive one.
- For r >> d, |**E**| varies inversely as r^3.

An Electric Dipole in an External Field

- If an electric dipole is placed in a uniform external electric field **E**, there is zero net force on the dipole (Why?).
- There is a net torque on the dipole: $\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

(Note that because the net force is zero, this expression for the torque is correct for **any** choice of coordinate origin.)

- In a non-uniform electric field, a dipole suffers both a torque and a net force. (Why?)

Electric Potential due to a Dipole

- At radii r >> d, the electric potential due to a dipole $\vec{p} = (|Q|d)\hat{k}$ centered at the origin is given to a very good approximation by:

$$\boxed{V_{dipole}(r,\theta) = \frac{|Q|}{4\pi\varepsilon_o}\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{p\cos\theta}{r^2}}$$

- Notes:
  - (1) Here the reference is V → 0 as r → ∞.
  - (2) θ is the colatitude (angle between **r** and +z)
  - (3) For given r, how does V vary with θ?

Using the Potential to Find the Field

$$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})$$

In spherical polar coordinates,

$$\vec{\nabla} f \equiv \hat{r}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}$$

$$\Rightarrow \boxed{\vec{E}(r,\theta) = \frac{p}{4\pi\varepsilon_o}\left[\hat{r}\left(\frac{2\cos\theta}{r^3}\right) + \hat{\theta}\left(\frac{\sin\theta}{r^3}\right)\right]}$$

Potential Energy of an Assembly of Point Charges

- Using Coulomb's law, the work required to bring two charges near each other (from infinite separation) is:

$$W_{\substack{to\\assemble}} = \Delta(PE) = \frac{1}{4\pi\varepsilon_o}\frac{Q_1 Q_2}{|\vec{r}_2 - \vec{r}_1|} \equiv \frac{1}{4\pi\varepsilon_o}\frac{Q_1 Q_2}{r_{12}}$$

- With infinite mutual separation as the zero for PE, we can generalize to N charged particles:

$$\boxed{PE = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{N}\sum_{j=i+1}^{N}\frac{Q_i Q_j}{|\vec{r}_j - \vec{r}_i|} \equiv \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{N}\sum_{j=i+1}^{N}\frac{Q_i Q_j}{r_{ij}}}$$

Lightning Rods

- These are among the many useful inventions of Benjamin Franklin.
- They work because:
  - (1) The magnitude of the electric field immediately surrounding a conducting object is greatest where the object is "sharpest" (has smallest radius of curvature).
  - (2) Air breaks down (ionizes) and provides a conducting path when the field exceeds a certain critical value (about 3 x 10^6 N/C = 3 x 10^6 V/m).