Reese Ch. 18

(Sections 6-11 = pp. 813-825)

- Capacitors
- Series & Parallel Combinations of Capacitors
- Energy Storage in a Capacitor
- Electrostatics in Dielectrics (= Insulators)
- Use of Dielectrics in Capacitors
- Dielectric Breakdown

Capacitors: I

- A capacitor is a circuit element that can store (separated) charge and electric potential energy.
- In most capacitors, the separated opposite electric charges are equal in magnitude.
- Ideally, once a capacitor is "charged" it can "hold its charge" indefinitely. Real capacitors "leak."
- The capacitance C of a capacitor is the ratio of the magnitude of separated charge to the (magnitude of the) potential difference:

$$\boxed{C \equiv \frac{|Q|}{|V|}}$$

Capacitors: II

- NOTES:
  (1) The capacitance is positive by definition.
  (2) Capacitance unit = farad (F):

  1 farad = 1 coulomb/volt

- In a "vacuum capacitor" (one that is free of dielectric materials), the capacitance is determined by the size, shape, and separation of the plates.

Capacitors: III

EXAMPLES:

(1) Parallel-plate capacitor: Each plate area = A; plates are separated by d<<A:

$$\boxed{C = \varepsilon_o \frac{A}{d}}$$

(2) Concentric spheres:

$$\boxed{C = \varepsilon_o \frac{4\pi r_1 r_2}{(r_2 - r_1)}}$$

(3) Cylindrical capacitor: (radii a & b, length l)

$$\boxed{C = \varepsilon_o \frac{2\pi b\ell}{b\ln(b/a)}}$$

Series and Parallel Combinations of Capacitors

- When capacitors are connected in parallel, they have the same voltage but the stored charge is the sum of the individual stored charges, so

$$\boxed{C_{\substack{equiv\\(parallel)}} = C_1 + C_2}$$

- When capacitors are connected in series, they store equal charges, but the voltage across the assembly is the sum of individual voltages, so

$$\boxed{C_{\substack{equiv\\(series)}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}}$$

Energy Storage in a Capacitor: I

- If given the opportunity to "discharge," a charged capacitor can do a lot of work (wreak a lot of havoc!). Evidently a charged capacitor stores potential energy in the separated charges.
- Taking a completely discharged capacitor as the reference state and carefully calculating the work necessary to separate the charges, we find:

$$\boxed{PE = \int_0^Q V(q)dq = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C} = \frac{1}{2}CV^2}$$

Energy Storage in a Capacitor: II

- It is possible to express the energy stored in a capacitor as a volume integral of <u>electric energy density</u>: (Why do it?)

$$\boxed{PE = \int_{\substack{all\\space}} \left(\frac{1}{2}\varepsilon_o \vec{E}\bullet\vec{E}\right) d(volume) \equiv \int_{\substack{all\\space}} \left(\frac{1}{2}\varepsilon_o E^2\right) dV}$$

- This is easily shown for a parallel-plate capacitor. (Try it!) It turns out to be a valid result for **any** vacuum capacitor.

Electrostatics in Dielectrics (=Insulators)

- The vacuum equations that we have been using can also be applied in the presence of many dielectric materials provided that we replace ε_o by ε, the <u>permittivity of the material</u>. A dielectric material is often characterized by its <u>dielectric constant κ</u>:
- Notice that κ is dimensionless.

$$\boxed{\kappa \equiv \frac{\varepsilon}{\varepsilon_o}}$$

Some κ values:
Rubber: ~3; Pyrex: 5.6 ; H_2O: 80

Use of Dielectrics in Capacitors

- When a vacuum capacitor is "filled" with a dielectric material, its capacitance is **multiplied** by κ, implying **more energy stored per volt**.
- This is due to **polarization** of the dielectric, which produces a bound-charge surface density at each plate. Each bound-charge surface density is opposite in sign to the adjacent "free-charge surface densities":

$$\boxed{\sigma_{bound} = -\left(\frac{\kappa - 1}{\kappa}\right)\sigma_{free}}$$

Dielectric Breakdown

- If a dielectric material is subjected to a sufficiently strong electric field (typically a few million to a few hundred million volts per meter), the atoms and molecules in the material can lose their outer electrons.
- These liberated electrons can collide with other atoms and molecules, knocking loose still more electrons. The result is that the material temporarily becomes a conductor.
- **Lightning** is the meteorological dielectric breakdown of air.