

Reese Ch. 21

(Sects. 4-5 = pp. 965-971)

- 3 Important Theorems Involving "Del"
- Integral Form of Maxwell's Equations
- Differential Form of Maxwell's Equations
- · Maxwell's Equations away from Charges and Currents
- Propagating Wave Solutions

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Important Theorems Involving Del: I

The gradient theorem:

$$f(\vec{r}_2) - f(\vec{r}_1) = \int_{\substack{\text{any path} \\ (1 \to 2)}} \left[\vec{\nabla} f(\vec{r}) \right] \cdot d\vec{r}$$

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Important Theorems Involving Del: II

The divergence theorem:

$$\bigoplus_{\substack{\text{closed} \\ \text{surface}}} \vec{u}(\vec{r}) \cdot d\vec{S} = \bigoplus_{\substack{\text{enclosed} \\ \text{volume}}} \left[\vec{\nabla} \cdot \vec{u}(\vec{r}) \right] dV$$

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Chapter 21

Important Theorems Involving Del: III

Stokes' theorem (= the curl theorem):

$$\oint_{\text{closed curve}} \vec{u}(\vec{r}) \cdot d\vec{r} = \iint_{\text{any bounded surface (open)}} \left[\vec{\nabla} \times \vec{u}(\vec{r}) \right] \cdot d\vec{S}$$

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Chapter 21

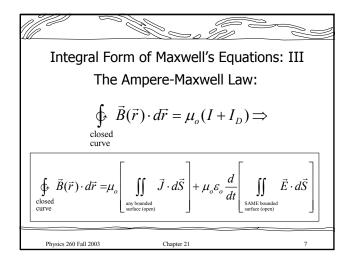


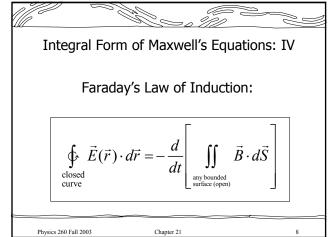
Integral Form of Maxwell's Equations: I

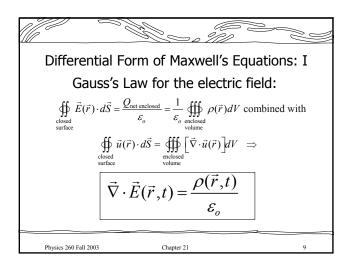
Gauss's Law for the electric field:

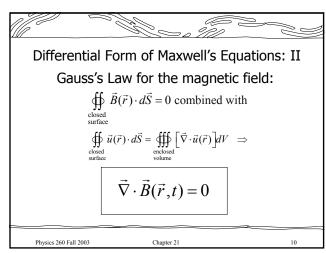
$$\bigoplus_{\substack{\text{closed} \\ \text{surface}}} \vec{E}(\vec{r}) \cdot d\vec{S} = \frac{Q_{\text{net enclosed}}}{\mathcal{E}_o} = \frac{1}{\mathcal{E}_o} \bigoplus_{\substack{\text{enclosed} \\ \text{volume}}} \rho(\vec{r}) dV$$

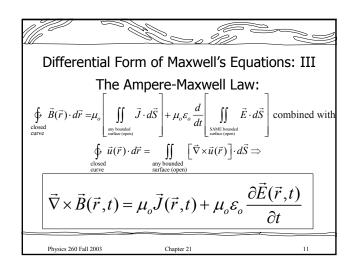
Physics 260 Fall 2003 Chapter 21 5

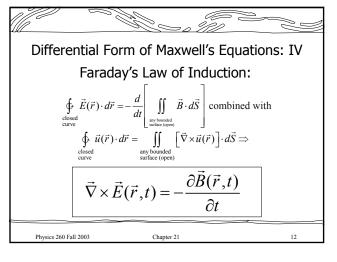


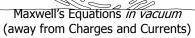












$$\vec{\nabla} \cdot \vec{E}(\vec{r}, t) = 0$$

$$\vec{\nabla} \cdot \vec{B}(\vec{r}, t) = 0$$

$$\nabla \times \vec{B}(\vec{r},t) = \mu_o \varepsilon_o \frac{\partial \vec{E}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$$

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Chapter 21

Propagating Wave Solutions: I

Notice that the permittivity ϵ_0 and the permeability μ

Notice that the permittivity ϵ_o and the permeability μ_o appear in the third equation as the product $\mu_o\epsilon_o$. What is its SI value?

$$\mu_o \varepsilon_o = (4\pi \times 10^{-7} \frac{T \cdot m}{A}) \times (8.854 \times 10^{-12} \frac{C^2}{N \cdot m^2})$$

$$\Rightarrow \mu_o \varepsilon_o = 0.1113 \times 10^{-16} \frac{s^2}{m^2} \equiv \frac{1}{c^2}$$

where the constant $c = (\mu_o \varepsilon_o)^{-1/2} = 2.998 \times 10^8 \frac{m}{s}$

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Propagating Wave Solutions: II I. Gauss E $\vec{\nabla} \cdot \vec{E}(\vec{r},t) = 0$ II: Gauss B $\vec{\nabla} \cdot \vec{B}(\vec{r},t) = 0$ III: Ampere-Maxwell $\vec{\nabla} \times \vec{B}(\vec{r},t) = \frac{1}{c^2} \frac{\partial \vec{E}(\vec{r},t)}{\partial t}$ IV: Faraday $\vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$

Chapter 21

Propagating Wave Solutions: III

- These equations allow many solutions let's look for the simplest of them.
 - Assume that the fields only depend on ONE spatial coordinate, which we choose to be x.
 - Assume that the electric field has zero x and z components: $\boxed{\vec{E}(\vec{r},t) = \hat{j}E_{_{Y}}(x,t)}$
 - Assume that the magnetic field has zero x and y components:

 $\vec{B}(\vec{r},t) = \hat{k}B_z(x,t)$

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Chapter 21

16



I. Gauss E $\Rightarrow 0 = 0$

 $\Rightarrow 0 = 0$ II: Gauss B

 $1 \partial E$ ∂B, III: Ampere-Maxwell

 ∂B_z IV: Faraday

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Propagating Wave Solutions: V

Differentiate III with respect to t and IV with respect to x:

III: $\frac{\partial^2 B_z}{\partial t \partial x} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2}$

IV:

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Propagating Wave Solutions: VI

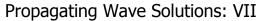


BINGO! $\Rightarrow \frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2}$

This is "the wave equation", for which the solution is $E_{v}(x,t) = f(x-ct) + g(x+ct)$

where f(u) & g(u) are arbitrary functions.

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If we choose g=0, then we get a rightward

moving wave:
$$\vec{E}(x,t) = \hat{j}E_y(x,t) = \hat{j}f(x-ct)$$
 and $\vec{B}(x,t) = \hat{k}B_z(x,t) = +\hat{k}c^{-1}f(x-ct)$

If we choose f=0, then we get a leftward moving wave:

$$\vec{E}(x,t) = \hat{j}E_y(x,t) = \hat{j}g(x+ct)$$
 and $\vec{B}(x,t) = \hat{k}B_z(x,t) = -\hat{k}c^{-1}g(x+ct)$

NOTE: **E x B** gives the direction of propagation

Chapter 21

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