Reese Ch. 25, Part A1

(Sects. 1-4 = pp. 1149-1160)

- Reference Frames & Galilean Relativity
- Galilean Transformation Equations between Two Inertial Frames
  - Transformation of Time and of Coordinates
  - Transformation of Velocities
  - Transformation of Acceleration
- Invariance of Speed of Light is Incompatible with Galilean Relativity
- Time Dilation

Reference Frames

- A **reference frame** consists of a coordinate system and a distributed set of synchronize clocks.
- An **inertial (reference) frame** is a reference frame in which Newton's first law (the law of inertia) is satisfied.
  - Both Galilean relativity and special (Einsteinian) relativity relate observations made by "inertial observers."
  - In special relativity, as in Newtonian physics, any inertial observer is in uniform translation motion with respect to any other inertial observer.

Galilean Relativity: I

- The basic equations of a relativity theory are **transformation equations**, which express the time and space coordinates of any "**point event**" in one reference frame ***in terms of*** the time and space coordinates of the **same event** in another reference frame.
- To obtain a definite set of equations, we make the (standard) assumption that frame S′ moves with constant velocity $\vec{v} = v\hat{i}$ with respect to frame S.

Galilean Relativity: II

- We also assume that:
  - the x′ axis of S′ coincides with the x axis of S
  - the y′ & y axes and the z′ & z axes are parallel
  - the point event defined by the momentary coincidence of the origins O and O′ has t=0.
- The basic Galilean transformation equations are then:

$$\boxed{\begin{aligned} t' &= t \\ x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned}}$$

Galilean Relativity: III

- We can derive the velocity transformation equations from the coordinate transformation equations. The results are: (These were used extensively in solving "relative motion" problems in PHYS 240.)

$$u_x' \equiv \frac{dx'}{dt'} = u_x - v$$

$$u_y' \equiv \frac{dy'}{dt'} = u_y$$

$$u_z' \equiv \frac{dz'}{dt'} = u_z$$

Galilean Relativity: IV

- The acceleration transformation equations are obtained by taking one more time derivative. We find that a particle's acceleration in frame S' just equals that in frame S. (Would this still be true if one of the frames were noninertial?)

$$a_x' \equiv \frac{du_x'}{dt'} = a_x - \frac{dv}{dt} = a_x$$

$$a_y' \equiv \frac{du_y'}{dt'} = a_y$$

$$a_z' \equiv \frac{du_z'}{dt'} = a_z$$

Invariance of Speed of Light

- Maxwell's equations predict a speed for electromagnetic waves that matches the measured speed of light.
- By analogy with sound waves, Maxwell and his contemporaries assumed that light waves must travel in some "medium" and that the speed predicted in his equations is the speed of electromagnetic waves *with respect to that medium (called the "ether")*.
- BUT . . . many experiments have shown that light travels with the same speed with respect to EVERY inertial observer, regardless of the motion of the light source or the observer! This means Galilean relativity is simply wrong when high speeds are involved.

The Postulates of Special Relativity

In 1905, Albert Einstein created a "new relativity" (special relativity) on the basis of two postulates:

(1) Not just the laws of mechanics, but all of the laws of physics look the same in every inertial frame of reference.

(2) Among these laws are Maxwell's equations, which implies that speed of electromagnetic waves has the same value in every inertial frame of reference.

Immediate Consequence: Time Dilation

- On the basis of these two postulates and the assumption (to be validated in the next lecture) that relative motion of two inertial frames does NOT affect lengths which are perpendicular to the relative motion, we can show that any moving clock "runs slow" when compared with the series of clocks past which it is moving. (Yikes!!)
- To be more precise, if two point-events are measured in S' to occur at the same place and a time interval $\Delta t'$ apart, the observer S will measure the time interval as:

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

Reese Ch. 25, Part A2

(Sects. 5-10 = pp. 1160-1171)

- Transformation of Lengths:
  - Transverse Lengths are Unaffected
  - Longitudinal Lengths are Contracted (Moving meter sticks are shortened.)
- Lorentz Transformation Equations for Time and Space Coordinates
- Relativity of Simultaneity
- The Twin Paradox

Transformation of Lengths: I

- We consider the possibility that we mistakenly assumed that the transversely oriented "laser clocks" had the same length when moving as at rest.
- If we assume that transverse lengths of moving objects are stretched, we soon find a logical contradiction.
- If we assume that transverse lengths of moving objects are compressed, we also find a logical contradiction.
- Inescapable conclusion: Transverse dimensions are unaffected by motion.

Transformation of Lengths: II

- The time dilation effect ("moving clocks run slow") implies that a radioactive particle "lasts longer" as reckoned in the "lab" reference frame S through which it is moving than it does in its own rest frame S'.
- Since the two frames are in relative motion with an agreed-upon relative speed v, this means observer in S' must reckon a smaller value than S does for the distance between the lab location where the particle was produced and the lab location where the particle decayed. In other words, the (longitudinal) lab distance is "contracted" when observed from the frame S'.

Transformation of Lengths: III

- In summary, a object which is a cube of length L_o in its rest frame and which is oriented with its edges parallel to the coordinate axes. . .

  (1) . . . also has transverse dimensions equal to L_o as reckoned in a reference frame through which it is moving, but . . .

  (2) has a longitudinal dimension (as reckoned in a reference frame through which it is moving) that is smaller by the factor γ:

  $\boxed{L = L_o/\gamma}$ where $\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

  Hence we use the phrase **length contraction**.

The Lorentz Transformation Equations: I

- In seeking the special-relativistic equations that replace the Galilean transformation equations, we want to preserve the feature that unaccelerated motion in frame S corresponds to unaccelerated motion in frame S'. (Why?) This implies that the transformation equations be **linear**.
- The fact that transverse lengths are unaffected by motion suggests that we keep the Galilean transformation equations for y and z:

$$\boxed{\begin{aligned} y' &= y \\ z' &= z \end{aligned}}$$

The Lorentz Transformation Equations: II

- For the transformation of the longitudinal space coordinate (x) and the time coordinate (t), we can expect: x' = Ax+Bt. But from the facts that (1) the origin of S' coincides with the origin of S at S-time = 0 (that is, t = 0) and (2) the origin of S' moves rightward through S at speed v, we find B = -Av.
- Thus we have x' = Ax-Avt=A(x-vt). We know that for v<<c, A→1. (Why?)
- We also know that x = A(x'+vt'). (Why?)

Lorentz Transformation Equations: III

- By thinking carefully about length contraction, we can conclude that A=γ. Then one can use $x' = \gamma(x - vt)$ and $x = \gamma(x' + vt')$ to find an equation for t' in terms of x and t: $t' = \gamma(t - \frac{vx}{c^2})$
- In preparing to summarize the transformation equations, it is helpful to introduce the symbol $\boxed{\beta \equiv \frac{v}{c}}$ for the ratio of the relative speed v to the speed of light.

Lorentz Transformation Equations: IV

- The Lorentz transformation from S to S′ is:

$$\boxed{\begin{aligned} x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \\ ct' &= \gamma(ct - \beta x) \end{aligned}}$$

where

$$\boxed{\beta \equiv \frac{v}{c} \text{ and } \gamma \equiv (1-\beta^2)^{-\frac{1}{2}}}$$

Lorentz Transformation Equations: V

- For the inverse Lorentz transformation from S′ to S, we simply replace β by –β:

$$\boxed{\begin{aligned} x &= \gamma(x' + \beta ct') \\ y &= y' \\ z &= z' \\ ct &= \gamma(ct' + \beta x') \end{aligned}}$$

The definitions of β and γ are unchanged.

The Relativity of Simultaneity

- In Newtonian physics, two events that are simultaneous in one reference frame are also simultaneous in every other reference frame: simultaneity is a "Galilean absolute."
- The Lorentz transformation from t to t′ makes it clear that two events which are simultaneous in frame S are not necessarily simultaneous in frame S′:

$$c\Delta t' = \gamma(c\Delta t - \beta\Delta x)$$

$$c\Delta t = \gamma(c\Delta t' + \beta\Delta x')$$

- In other words, simultaneity is NOT a "Lorentz absolute."

The Twin "Paradox"

- The Lorentz transformation equations maintain a complete reciprocity between the frames S and S′. Through observations <u>in either frame</u>, one can conclude that the other frame's "moving clocks" run slow and its "moving (longitudinal) meter sticks" are contracted.
- Thus it is surprising to learn that a careful analysis of a hypothetical high-speed long-distance round trip by one member of a pair of twins predicts a difference in the twins' ages at the end of the trip! However, while unexpected, this result is <u>not a real paradox</u>: rational observers in both frames can do analyses that agree. What's more, experiments support the prediction!

Reese Ch. 25, part A3

(Sects. 11-15 = pp. 1171-1180)

- Lorentz Transformation of Velocity
  (aka Relativistic Velocity Addition)
- Observing (Measuring) vs. Seeing
  - Apparent Shape & Color of High-Speed Objects
  - The Optical Illusion of Superluminal Speed
- Relativistic Doppler Effect
  - Radially Receding or Approaching Source
  - Passing Source ("Transverse Doppler Effect")
  - General Case ("Oblique View")

Lorentz Transformation of Velocity: I

- Suppose that observers S and S′ both keep track of the motion of a particle P.
- Observer S determines that the particle's position as a function of time t is given by:

$$\boxed{\vec{r}_P(t) = \hat{i}x_P(t) + \hat{j}y_P(t) + \hat{k}z_P(t)}$$

- Observer S′ determines that the particle's position as a function of time t′ is given by:

$$\boxed{\vec{r}_P^{\,'}(t') = \hat{i}\,'x_P^{'}(t') + \hat{j}\,'y_P^{'}(t') + \hat{k}\,'z_P^{'}(t')}$$

- What **velocities** do the observers ascribe to P?

Lorentz Transformation of Velocity: II

- According to S, the particle's velocity at time t is:

$$\boxed{\vec{u}_P(t) \equiv \frac{d\vec{r}_P(t)}{dt}}$$

- According to S′, the particle's velocity at time t′ is:

$$\boxed{\vec{u}_P^{\,'}(t') \equiv \frac{d\vec{r}_P^{\,'}(t')}{dt'}}$$

- How are the velocity components related?

Lorentz Transformation of Velocity: III

- To relate the velocity components obtained in the two different frame, (1) recall that each velocity component is the limit of a ratio such as Δx/Δt, and (2) Lorentz-transform the space and time intervals:

$$\Delta x' = \gamma\left(\Delta x - v\Delta t\right)$$

$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

Lorentz Transformation of Velocity: IV

- This process provides the desired Lorentz transformation of velocity components.
- Notice carefully the denominators in these equations.

$$u_x'(t') = \frac{u_x(t) - v}{1 - \frac{v u_x(t)}{c^2}}$$

$$u_y'(t') = \frac{u_y(t)}{\gamma\left[1 - \frac{v u_x(t)}{c^2}\right]}$$

$$u_z'(t') = \frac{u_z(t)}{\gamma\left[1 - \frac{v u_x(t)}{c^2}\right]}$$

Lorentz Transformation of Velocity: V

- To get the inverse transformation, wade through algebra only to find that you could simply have substituted –v for v everywhere!

$$u_x(t) = \frac{u_x'(t') + v}{1 + \frac{v u_x'(t')}{c^2}}$$

$$u_y(t) = \frac{u_y'(t')}{\gamma\left[1 + \frac{v u_x'(t')}{c^2}\right]}$$

$$u_z(t) = \frac{u_z'(t')}{\gamma\left[1 + \frac{v u_x'(t')}{c^2}\right]}$$

Observing (Measuring) vs. Seeing: I

- So far we have avoided the important distinction between observing and seeing.

  We can only observe by gathering and compiling the records made by the horde of individuals/instruments who collectively constitute "the observer S." But individual humans are accustomed to using their sense of sight and inferring what's happening on the basis of what they see.
- We need to examine this distinction!

Observing (Measuring) vs. Seeing: II

- Light-travel time and apparent shape of an an extended object:
  - An individual's visual system (obviously) processes together the photons that are received together.
  - The finite speed of light implies that (1) We now see a stationary object at distance D not as it is now, but as it was a time D/c ago; and (2) For extended objects, the light-travel time D/c is different for different parts of the object.
  - The consequence is that objects moving at a substantial fraction of the speed of light appear to bend and rotate as we watch them!

Observing (Measuring) vs. Seeing: III

- The Doppler shift and the apparent color of a moving object.
  - For a monochromatic light source that is directly **approaching** us, the frequency of the light that we receive is higher than the frequency emitted (I.e., higher than the frequency as measured by someone in the source's "rest frame.") This is termed a **blueshift.**
  - For a source that is directly **receding** from us, the frequency of the light that we receive is lower than the frequency emitted (I.e., lower than the frequency as measured by someone in the source's "rest frame.") This is termed a **redshift.**

Observing (Measuring) vs. Seeing: IV

- The distinction between observing and seeing is responsible for the **apparently superluminal speeds** of material in some quasars.
- As derived in the text, luminous material ejected from a distant object at speed βc and traveling obliquely toward us at an angle θ (measured in our frame!) is seen to move "across the sky" at an angular rate which (when multiplied by the distance of the object) suggests an apparent speed $\beta_{apparent}c$:

$$\beta_{apparent} = \frac{\beta \sin\theta}{1-\beta\cos\theta}$$

- There are cases in which the values of β and θ imply $\beta_{apparent} > 1$!! ***Cool!!***

Relativistic Doppler Effect: I

- The Doppler effect refers to the fact that when source and observer are in relative motion, the received frequency of the waves is different from the source frequency.
- According to pre-Einsteinian (I.e., Galilean) relativity, the Doppler "shift" is due solely to the fact that the travel time for each wave crest is shorter (longer) than the travel time for the preceding crest if the source is approaching (receding from) the receiver (observer).
- The (special-) relativistic Doppler shift results from not only decreasing (or increasing) light-travel times **BUT ALSO from time dilation** (moving clocks run slow).

Relativistic Doppler Effect: II

- The analysis presented in the text shows that for a source moving directly away (at speed v) from the receiver, the received frequency is given by:

$$f_{received} = f_{source}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

(if source is receding)

Relativistic Doppler Effect: III

- For a source moving directly toward the receiver at speed , the received frequency is given by:

$$f_{received} = f_{source}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

(if source is approaching)

Relativistic Doppler Effect: IV

- If the source is moving at right angles (transversely) to the line of sight from the receiver to the source , the received frequency is given by:

$$f_{received} = f_{source}\sqrt{1-\frac{v^2}{c^2}} = \frac{f_{source}}{\gamma}$$

(transverse Doppler effect)

Relativistic Doppler Effect: V

- The general formula for the relativistic Doppler effect includes the previous formulae as special cases. The waves which are emitted when the source is moving at an angle θ to the inward radial direction are received (later, of course!) at a frequency given by

$$f_{received} = \frac{f_{source}\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}\cos\theta} = \frac{f_{source}}{\gamma\left(1-\beta\cos\theta\right)}$$