Reese Ch. 25, Part B1

(Sects. 16-19 = pp. 1180-1188)

- Force and Momentum in Special Relativity
- Work-Energy Theorem & Relativistic Energy
- Implications of Mass-Energy Equivalence
- Spacetime and Spacetime Diagrams
  - Invariance of Spacetime Interval between Events
  - Timelike, Spacelike, and Lightlike Intervals
  - Worldlines

Relativistic Force and Momentum: I

- By analyzing a hypothetical elastic scattering experiment that has a specific symmetry, we find that the vector sum $\sum_i m_i \vec{u}_i$ for a system is <u>NOT conserved</u> in special relativity. The most similar vector quantity which IS conserved for any isolated system and which is adopted as the definition of relativistic total momentum is:

$$\boxed{\vec{P}_{system} \equiv \sum_i \frac{m_i \vec{u}_i}{\sqrt{1-\frac{u_i^2}{c^2}}}}$$

Relativistic Force and Momentum: II

- With the momentum of a particle given by

$$\boxed{\vec{p} \equiv \frac{m\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}} \equiv \gamma_u m\vec{u}}$$

the special-relativistic generalization of Newton's Second Law is: $\boxed{\vec{F}_{net} = \frac{d\vec{p}}{dt}}$

- Experiments support this form of Newton's Second Law. For example, a charged particle moving in electric and magnetic fields satisfies: $\boxed{\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{u} \times \vec{B}\right)}$

Relativistic Force and Momentum: III

- Notice that a particle moving at a speed near the speed of light is much more difficult to accelerate than a slow-moving particle:

$$\boxed{\frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\gamma_u m\vec{u}\right) = \left(\frac{d\gamma_u}{dt}\right) m\vec{u} + \gamma_u m \frac{d\vec{u}}{dt} = \vec{F}_{net}}$$

- Furthermore, the acceleration vector does not necessarily point in the same direction as the force!

$$\boxed{\frac{d\vec{u}}{dt} = \frac{\vec{F}_{net}}{\gamma_u m} - \left(\frac{d\left(\ln \gamma_u\right)}{dt}\right)\vec{u}}$$

Work-Energy Theorem: I

- The special relativistic definition of work is exactly the same as the pre-SR definition:

$$\boxed{W_{i\to f} \equiv \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\bullet d\vec{r} \Leftrightarrow dW_{i\to f} \equiv \vec{F}\bullet d\vec{r}}$$

- It is an exercise in calculus and algebra to use this definition to show that:

$$dW_{net} \equiv \vec{F}_{net}\bullet d\vec{r} = d\left(\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}}\right)$$

Work-Energy Theorem: II

- This means that the SR version of the CWE theorem reads:

$$\boxed{\frac{mc^2}{\sqrt{1-\frac{u_f^2}{c^2}}} - \frac{mc^2}{\sqrt{1-\frac{u_i^2}{c^2}}} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{net}\bullet d\vec{r}}$$

- If we consider a particle which starts from rest, we conclude that the SR expression for kinetic energy must be $\boxed{KE = (\gamma_u - 1)mc^2}$

Relativistic Energy and Momentum: I

- Notice that the kinetic energy is the difference between a speed-dependent term and one that depends only upon the mass:

$$KE = \gamma_u mc^2 - mc^2$$

- Rearranging this equations, we define the relativistic total energy E as the sum of the kinetic energy KE and the quantity mc^2 (the "rest energy"):

$$\boxed{E \equiv KE + mc^2 = \gamma_u mc^2}$$

Relativistic Energy & Momentum: II

- With a modest investment of algebraic work, two useful relationships between p and E can be obtained:

$$p = \frac{uE}{c^2} \leftrightarrow pc = \beta_u E$$

$$E^2 = (pc)^2 + (mc^2)^2$$

Mass-Energy Equivalence: I

- In pre-Einsteinian physics, scientists regarded conservation of energy and conservation of mass as two independent conservation laws. However, SR indicates (and experiments support the notion) that energy and mass are inextricably linked. In an isolated system of particles, the sum of the relativistic energies is conserved, but decays and reactions can occur in which the sum of the rest energies changes. There is a resulting change Q in the kinetic energy of the system:

$$\boxed{Q \equiv \sum_{final} KE - \sum_{initial} KE = \sum_{initial} mc^2 - \sum_{final} mc^2 = -c^2(\Delta M)}$$

Mass-Energy Equivalence: II

- In the popular culture, the equation $Q = -\Delta(Mc^2)$ is known as $E = mc^2$. It is arguably the most famous equation in science.
- The most spectacular examples of the partial conversion of mass to (kinetic) energy involve nuclear transformations. However, as far as we know the equation is universal. For example, in a typical exothermic chemical reaction, the total rest mass of the product chemicals is smaller than the total rest mass of the reagent chemicals (by something like 0.1-1 part in a billion).

Spacetime: I

- In Newtonian physics, the time interval Δt between two events is the same in all reference frames, so simultaneity is absolute. Furthermore, the spatial separation between two simultaneous events is the same in all reference frames.
- Special relativity does not incorporate either of these features, but there is a quantity which is the same in both reference frames. It is the <u>invariant spacetime interval</u> Δs:

$$\boxed{\Delta s^2 \equiv (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2}$$

- To prove $\Delta s' = \Delta s$, use the Lorentz transformation.

Spacetime: II

- There are three basic types of spacetime intervals:

  (1) **<u>Timelike intervals ($\Delta s^2<0$).</u>** There are reference frames in which both events happen at the same place. Event 2 occurs after event 1 in every frame, and in principle the events can be causally related.

  (2) **<u>Lightlike intervals ($\Delta s^2=0$).</u>** In every reference frame, a light signal (but no material particle) emitted at event 1 can reach event 2.

  (3) **Spacelike <u>intervals ($\Delta s^2>0$).</u>** There are reference frames in which the events happen at the same time but different places, and there are also reference frames in which event 2 <u>precedes</u> event 1! The events <u>cannot</u> be causally related.

Spacetime: III

- The world-line of a particle is the curve in spacetime that joins all the events for which the particle is present. In others words, it's the graph in the four-dimensional space (x,y,z,ct) of the particle's entire existence:

$$\text{worldine of particle P} = \{[x_P(t), y_P(t), z_P(t), ct]\}$$

- Any two points on the worldline of a particle of nonzero rest mass must be separated by a timelike interval. (Why?)

Reese Ch. 25, Part B2

(Sects. 20-21 = pp. 1188-1194)

- Special Relativity and Electromagnetism
  - Lorentz Force on a Moving Charge
  - A ***Magnetic Force*** in one frame may be an ***Electric Force*** in another frame!
  - Transformation Equations for Fields
- Introduction to the General Theory of Relativity
  - Status of Special Relativity vs. Status of GR
  - Basic Notions of GR
  - Experimental Evidence regarding GR
  - Difficulties and Future Prospects

Lorentz Force Law

- If both an electric field and a magnetic field are present, a charged particle that has electric charge q, instantaneous position **r**(t), and instantaneous velocity **u**(t) is subject to a force that is the vector sum of the electric force q**E**(**r**,t) and the magnetic force q[**u**(t) x **B**(**r**,t)]. This is called the **Lorentz force**:

$$\vec{F}(t) = q[\vec{E}(\vec{r},t) + \vec{u} \times \vec{B}(\vec{r},t)]$$

- Like other physical laws, the Lorentz force law is valid in all inertial frames.

Linkage of Electric & Magnetic Fields

- There are a couple of different ways to obtain the transformation equations for electric and magnetic field:
  (1) Use "charge invariance" and proceed from Maxwell's equations and the Lorentz transformation of space coordinates and time.
  (2) Use "charge invariance", the Lorentz force law, and the transformation equations for space, time, velocity, and acceleration.
- BUT the basic result that a purely electric (or purely magnetic field) in one reference frame transforms into both electric and magnetic fields can be seen without algebraic toil. Consider a particle moving parallel to a current-carrying wire . . . (Reese pp. 1188-1190)

Transformation Equations for the Fields

- With the standard relationship between frame S and S′, the field transformations are:

$$E'_{x'} = E_x$$
$$E'_{y'} = \gamma(E_y - vB_z)$$
$$E'_{z'} = \gamma(E_z + vB_y)$$

$$B'_{x'} = B_x$$
$$B'_{y'} = \gamma\left(B_y + \frac{vE_z}{c^2}\right)$$
$$B'_{z'} = \gamma\left(B_z - \frac{vE_y}{c^2}\right)$$

- *No need to memorize these!*

Status of Special Relativity vs. Status of General Relativity

- Since its invention (discovery?) in 1905, special relativity (SR) has passed many experimental tests. Today it is regarded as very likely to remain a feature of physical law far into our future (forever?). These days most physicists believe that any theory which is incompatible with SR is fundamentally wrong.
- The general theory of relativity (GR) was invented (discovered?) in 1907-1915. Since then it has passed several tests but humans have not yet found any opportunities to test it in extreme environments. Most physicists regard GR as an excellent classical theory of gravity, but there will be great difficulties in reconciling it with quantum mechanics.

Basic Notions of General Relativity

- Principle of Equivalence: At every point in a gravitational field it is possible to find a "locally inertial frame" such that within a sufficiently small neighborhood of the point, the laws of nature take the same form as in an unaccelerated frame far from any source of gravity.
- Locally inertial frames are freely falling frames.
- The spacetime of SR is "flat"; the spacetime of GR is "curved."
- Spacetime tells mass how to move; mass tells spacetime how to curve.

Observational Evidence on General Relativity

- Excess Precession of Perihelion of Mercury (existence known long before GR, but GR explained the excess precession ***without any adjustable parameters!***)
- Gravitational Deflection of Starlight (1919 total solar eclipse expedition)
- Gravitational Redshift (1960 publication by Pound and Rebka – an amazing "stairwell experiment")
- Radar Echo Delay Experiments (1968 publication by Shapiro et alia – delay in radar echoes from Venus)
- Orbital Decay of a Binary Pulsar (1978 Taylor et alia announce orbital period decrease that would be expected due to emission of gravitational radiation)

GR: Difficulties and Future Prospects

- Apparent Incompatibility with Quantum Mechanics.
  General relativity is a classical (non-quantum) theory. It appears that there are serious barriers to modifying general relativity to respect the requirements of quantum physics.
- GR and Time Travel? There are some theoretical studies (by Kip Thorne and others) which suggest that with the "help" of wormholes, it might be possible to travel backward in time. Most physicists find this prospect bizarre and troubling.
- The Search for Gravitational Waves. An ambitious and expensive project is underway to construct very sensitive detectors for gravity waves. For more information, see http://www.ligo.caltech.edu/.