Reese Ch. 26, Part B1

(Sect. 14 = pp. 1231-1235)

- A Light Beam Should Carry Momentum as Well as Energy
  - Predicted by Maxwell's Classical EM Theory
  - Also a Consequence of Treating Einstein's Photon as a Particle of Zero Rest Mass
- Compton Scattering
- Consequences of the Momentum of Light: From Solar Sails to Nuclear Physics

Classical Electromagnetic Theory

In Maxwell's electromagnetic (EM) theory:

- Light and radio are just two examples of an entire spectrum of possible EM radiation.
- The direction of propagation is given by the vector $\vec{S} \equiv \vec{E} \times \vec{B}$. (It's called the "Poynting vector.")
- In fact, the Poynting vector **IS** the energy flux density vector (energy per unit time per unit area) carried by a light beam.
- Maxwell's theory can be used to show that the momentum flux density vector (momentum per unit area per unit area) equals $\vec{S}/c$.

Particle Theory: Photons

- Einstein's 1905 theory of the photoelectric effect introduced the idea that monochromatic light of frequency ν consists of a shower of photons, each carrying energy E = hν.
- If we treat them as particles in special relativity, we must regard them as particles of zero mass. (Why?)
- Remember the special relativistic equation that connects mass, momentum, and total energy:

$$\boxed{E^2 = (pc)^2 + (mc^2)^2}$$

- Since m = 0, we find $\boxed{p = \frac{E}{c} = \frac{h\nu}{c}}$ for a photon.

Wave and Particle Theories Agree!

- Consider a monochromatic beam of light. Using a photon model for it, we say that the energy flux density equals the photon number flux (photons per unit area per second) *N* times hν. The momentum flux density equals *N* x p, or *N* hν/c.

  In other words, in the particle theory, we get the momentum flux density by dividing the energy flux density by the speed of light.
- This is precisely the same relationship as given by Maxwell's EM wave theory.
- However, there are many phenomena that require use of quantum theory. An important one is . . .

. . . Compton Scattering

- Compton scattering (discovered in 1923 by Arthur Compton when he bombarded graphite with X rays) is the elastic scattering of a photon by an electron.
- Compton observed that the scattered radiation has a longer wavelength than the incident radiation, and he showed that his data were well explained by a relativistic "billiard-ball" analysis, according to which

$$\boxed{\lambda' = \lambda + \frac{h}{mc}(1-\cos\theta)}$$

- Here m is the electron mass and θ is the angle through which the x-ray photon is scattered.

Compton Scattering: II

- In the equation on the previous slide, the quantity (h/mc) has units of length. It has a jargon name and symbol: the "Compton wavelength of the electron" λ_{Ce}, even though it is NOT actually a wavelength. It is much less than the diameter of an atom (though larger than the diameter an atomic nucleus).

$$\boxed{\lambda_{Ce} = 2.43\times10^{-12}\,\mathrm{m} = 0.00243\ \mathrm{nm}}$$

- Compton's analysis shows that the longer the incident wavelength, the smaller is the fractional change in wavelength: $\boxed{\dfrac{\lambda'-\lambda}{\lambda} = \dfrac{\Delta\lambda}{\lambda} = \dfrac{\lambda_{Ce}}{\lambda}(1-\cos\theta)}$

Consequences of the Momentum of Light: I: Solar Radiation Pressure

- The dust tails of comets are due to the momentum deposited on fine dust particles by solar radiation.
  - The sun's "radiation pressure" opposes its gravitational attraction for the particles.
  - For particle diameters than a few tenths of a micron, the outward force exceeds the gravitational pull and such particles are driven completely out of the solar system.
- Some visionaries have suggested that huge solar sails be used for interstellar voyages.
- Question: Which type of surface experiences a bigger force per unit area: a shiny one or a flat black one?

Consequences of the Momentum of Light: II: Inverse Compton Scattering

- What happens if a high-speed electron encounters a photon? If we apply the conservation laws for momentum and momentum, we find that elastic scattering can transfer lots of energy from the electron to the photon. Although it is still just elastic scattering between an electron and a photon, in cases where the photon gains energy, we refer to it an "inverse" Compton scattering. Max($h\nu'$) $\approx 4\gamma_e^2(h\nu)$!!!
- This effect is used in experiments that Professor Whisnant is involved with at Brookhaven National Lab (on Long Island). Electrons which have an energy of 2.5 GeV ($\gamma_e \approx 4900$) are used to turn incident light of wavelength 350 nm (near UV) into gamma rays with energy 300 MeV! (See Dr. Whisnant if you'd like to know more about his research.)